

Problem 1: An RC circuit is to be designed to filter an audio signal with a DC bias so that the DC is eliminated. Since large filter transients are in general undesirable, the RC time constant should not be too large.

To create a more specific objective we consider the sum of:

1. A useful 40Hz signal $x_0(t) = \sin 40 \times 2\pi t$
2. A drift 0.1Hz signal $x_n(t) = \sin 0.1 \times 2\pi t$

We would like to select suitable R, C such that the RC circuit with transfer function

$$H(s) = \frac{RCs}{RCs + 1}$$

lets the useful signal through unchanged and stops the drift signal, as much as possible. For example, we would like to minimize the power of the error signal

$$H[x_0 + x_n] - x_0 = (1 - H)[x_0] + H[x_n]$$

As a first approximation, we can try to choose RC such that the steady state amplitudes of the two components are the same. These amplitudes can be computed from the Bode plots of $(1-H(s))$ and $H(s)$.

1. Use MATLAB to perform the necessary computations to obtain a coarse solution for the filter time constant RC;
2. select common values for a Resistor and a Capacitor to implement the filter;
3. simulate the time response of the filter and estimate approximately how long it would take to reach steady-state;
4. verify that the simulated signal amplitudes and time to steady-state are in agreement with the predictions from the Bode plots and the transient response of the filter.

Relevant MATLAB commands:

```
t=[0:.001:20]'; % time vector definition
x0=sin(40*6.28*t); % signal definition
xn=sin(0.1*6.28*t); % drift definition
RC=0.1; H=tf([1/RC 0],[1/RC 1]); % filter definition
Bode(1-H,H) % Bode plots
plot(t,x0+xn, t,x0,t,lsim(F,x0+xn,t)) % plot of noisy signal,
% clean signal, and filtered signal
plot(t,sim(F,x0+xn,t)-x0) % plot of error signal
```

Problem 2: Repeat Problem 1 for a second order filter that is a cascade of two first order RC filters. Here you can use $H_2 = H \cdot H$ to find the effective time constant. However, due to loading effects, simply cascading two first order filters does not produce the desired transfer function. Use Nodal/Loop analysis to find the transfer function of the general second order filter shown below, and based on the result make a reasonable selection of the two resistances and two capacitances to solve the problem.

