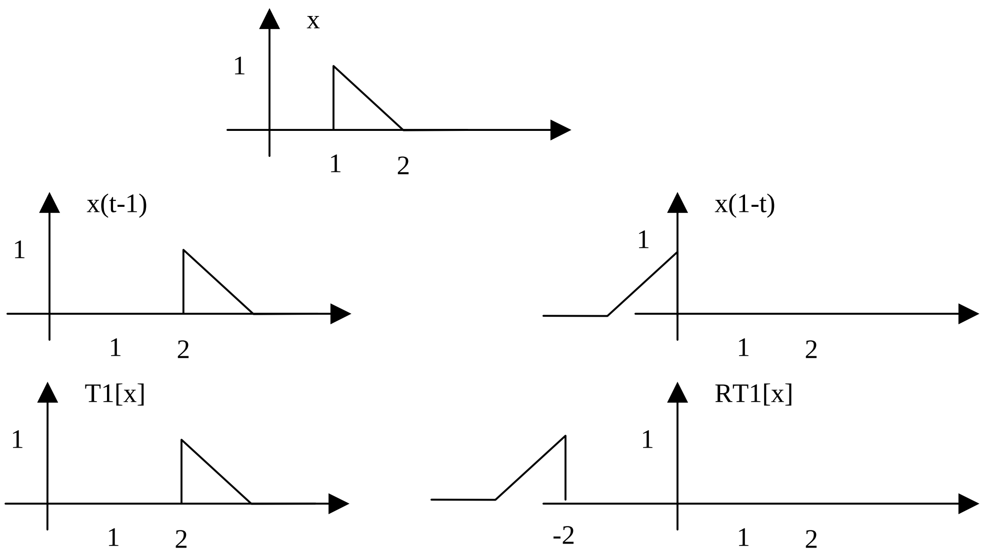
**NAME:____SOLUTIONS_____**

**EEE 203** **HW 1**

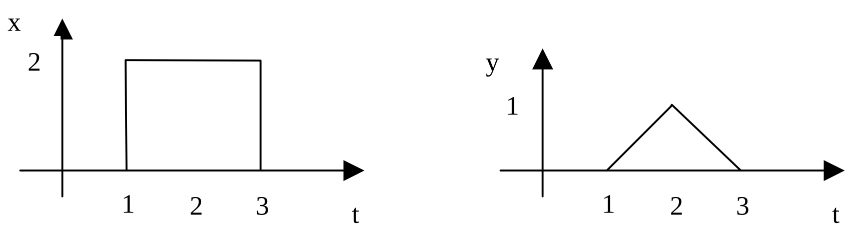
**Problem 1.**

Consider the signal *x(t)* whose graph is shown below. Sketch the following signals: *x(t-1), x(1-t)*, $RT_1[x]$, where R denotes the reflection operation and $T_{t0}$ denotes shift delay operation by t0.




**Problem 2.**

Describe the following signals in terms of elementary functions (δ, *u* ,*r*, …) and compute $\int_{-\infty}^{\infty} x(t)\delta(t-1)dt$ and $\int_{-\infty}^{\infty} y(t)\delta(t-1)dt$.




$$x(t) = 2u(t-1) - 2u(t-3)$$

$$y(t) = r(t-1) - 2r(t-2) + r(t-3)$$

$$\int_{-\infty}^{\infty} x(t)\delta(t-1)dt = \frac{x(1+) + x(1-)}{2} = 1$$

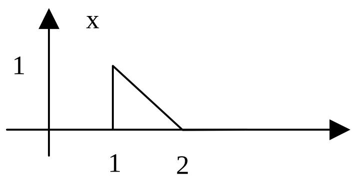
$$\int_{-\infty}^{\infty} y(t)\delta(t-1)dt = \frac{y(1+) + y(1-)}{2} = 0$$

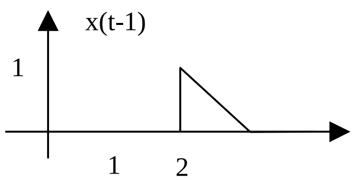
**NAME:_____________________**

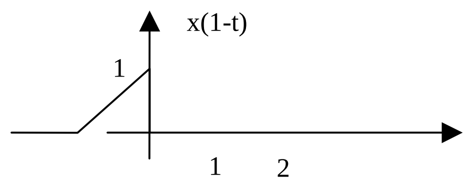
**EEE 203 TEST 1**

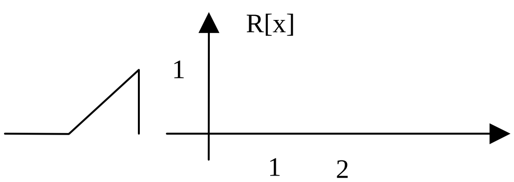
**Problem 1. (4pts)**

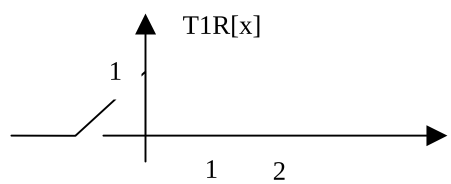
Consider the signal *x(t)* whose graph is shown below. Sketch the following signals: *x(t-1), x(1-t),* $T_1R[x]$, where R denotes the reflection operation and $T_{t0}$ denotes shift delay operation by t0.



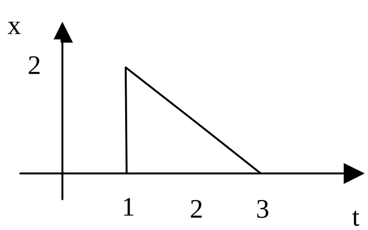



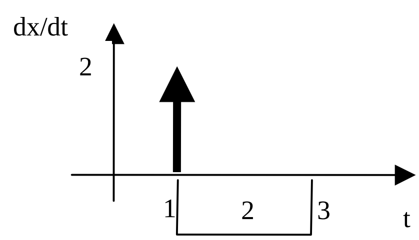




**Problem 2. (6pts)**

Describe the following signal in terms of elementary functions (δ, *u* ,*r*, …) and compute $\frac{dx(t)}{dt}$ and $\int_{-\infty}^{\infty} x(t+1)\delta(t-1)dt$ .




$$x(t) = 2u(t-1) - r(t-1) + r(t-3)$$

$$\frac{dx(t)}{dt} = 2\delta(t-1) - u(t-1) + u(t-3)$$

$$\int_{-\infty}^{\infty} x(t+1)\delta(t-1)dt = \int_{-\infty}^{\infty} x(1+1)\delta(t-1)dt$$

$$= 1\int_{-\infty}^{\infty} \delta(t-1)dt = 1$$

**NAME:____________________**

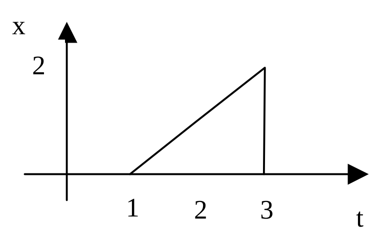
**EEE 203** **TEST 1**

**Problem 1. (6pts)**

Consider the signal *x(t)=u(t+1)-r(t)+r(t-1).* Sketch the following signals: *x(t), x(-t), x(t-2), x(2-t),* $T_{-2}R[x]$, where R denotes the reflection operation and $T_{t0}$ denotes shift delay operation by t0.

**Problem 2. (4pts)**

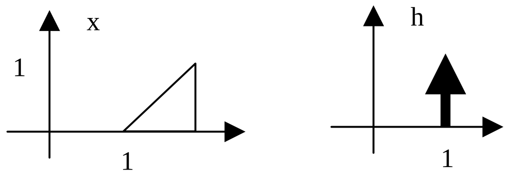
Describe the following signal in terms of elementary functions (δ, *u* ,*r*, …) and compute $\frac{dx(t)}{dt}$ and $\int_{-\infty}^{\infty} x(t+1)\delta(-t+1)dt$ .



**NAME:_____SOLUTIONS____**

**EEE 203** **HW 2**

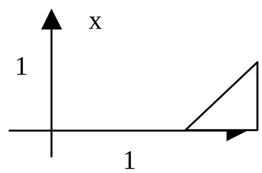
**Problem 1.** Compute the convolution $h*x$ when $x(t) = r(t)-r(t-1)-u(t-1)$, $h(t)=\delta(t-1)$.




($u(t)$ is the unit step)

Unfortunately, the x(t) shown is a delayed version of the expression: $x(t) = r(t-1)-r(t-2)-u(t-2)$.

$(h*x)(t) = [(\delta(t-1))*x](t) = x(t-1) = r(t-2)-r(t-3)-u(t-3)$



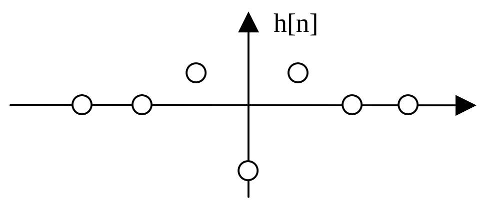

**Problem 2.**
Consider the filters:

**A.** $y[n] = x[n+1] - 2x[n] + x[n-1]$

**B.** $y(t) = \int_{-\infty}^{t+1} e^{t-\tau} x(\tau)\, d\tau$

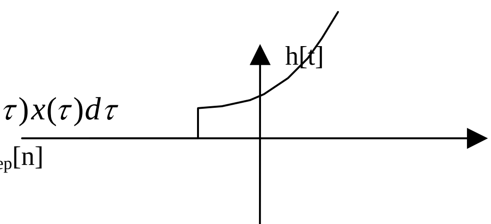
**1.** Find and graph their impulse responses.

A. $h[n] = \delta[n+1] - 2\delta[n] + \delta[n-1]$



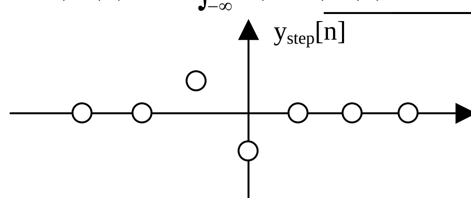

B. $y(t) = \int_{-\infty}^{t+1} e^{t-\tau} x(\tau)\, d\tau = \int_{-\infty}^{\infty} e^{t-\tau} u(t+1-\tau) x(\tau)\, d\tau = \int_{-\infty}^{\infty} h(t-\tau) x(\tau)\, d\tau$

$h(t) = e^t u(t+1)$

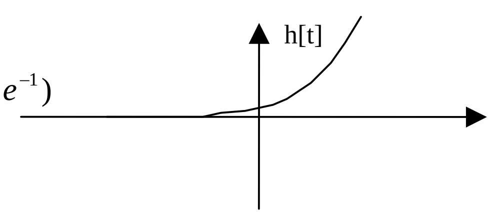



**2.** Find and graph their step responses.

A. $y_{\text{step}}[n] = u[n+1] - 2u[n] + u[n-1]$




B. $y_{step}(t) = \int_{-\infty}^{t+1} e^{t-\tau} u(\tau)\, d\tau = u(t+1) \int_{0}^{\cdots t+1} e^{t-\tau}\, d\tau = u(t+1)(e^t - e^{-1})$




**3.** Which filters are causal? (Justify)

A. Not causal, h(-1) is not zero.

B. Not causal, h(t) is nonzero in [-1,0).

**4.** Which filters are stable? (Justify)

A. Stable since it is FIR. h[n] is absolutely summable. ($\Sigma$|h[n]| = 1+2+1 = 4 < inf. )

B. Unstable since h(t) diverging implies that $\int_{-\infty}^{\infty} |h(t)|\, dt$ diverges.

**NAME:____SOLUTIONS_____**

**EEE 203** **TEST 2** **Wed. 9/27/06**

**30', closed books&notes**

**Problem 1.** Compute the convolution *h*x* when *x(t) = u(t) - u(t-1), h(t)=u(t).*

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau = \int_{-\infty}^{\infty} u(t-\tau)u(\tau)d\tau - \int_{-\infty}^{\infty} u(t-\tau)u(\tau-1)d\tau$$

$$= u(t)\int_0^t 1d\tau - u(t-1)\int_1^t 1d\tau = tu(t) - (t-1)u(t-1) = r(t) - r(t-1)$$

**Problem 2.**
Consider the filters:
**A**. $y[n] = x[n] - x[n-1]$
**B.** $y(t) = \int_{-\infty}^{t-1} x(\tau)d\tau$

**1.** Find and graph their impulse responses.
A. $h(n) = \delta(n) - \delta(n-1)$

B. $y(t) = \int_{-\infty}^{\infty} u(t-1-\tau)x(\tau)d\tau \Rightarrow h(t) = u(t-1)$

**2.** Which filters are causal? (Justify)
A. Causal because *h(n)* = 0, for all *n*<0.
B. Causal because *h(t)* = 0, for all *t*<0.

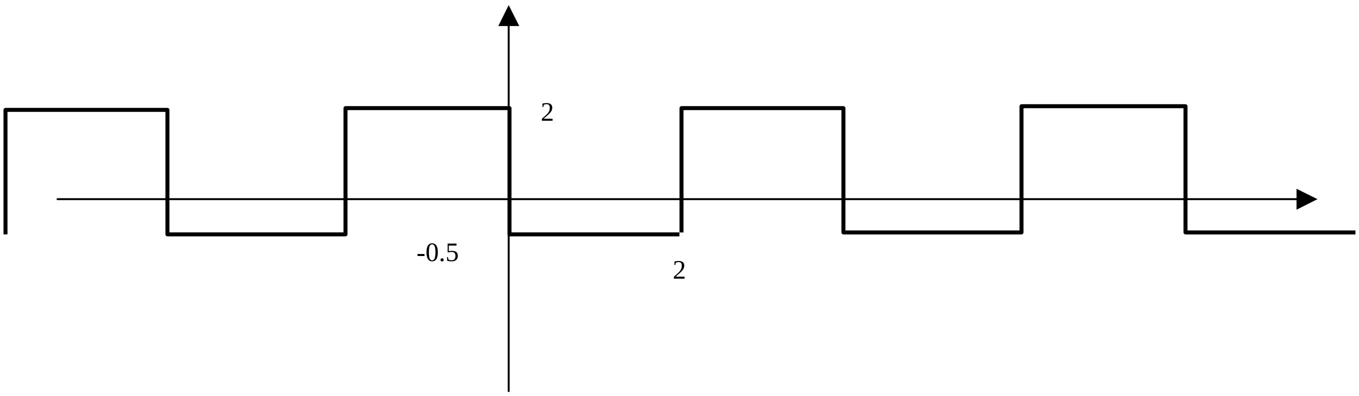
**3.** Which filters are stable? (Justify)
A. Stable because $\sum |h(n)| = 2 < \infty$

B. Unstable because $\int |h(t)|\,dt = \lim_{T\to\infty} \int_1^T 1dt$ , which diverges.

**NAME:____________________**

**EEE 203** **HW 3**

**Problem 1:** Let *x(t)* be the periodic signal shown in the figure below (square wave with offset). Compute the coefficients $a_k$ of the Fourier series expansion of *x(t).*




Let $x_0(t)$ be the standard square wave (in the Tables), with $T = 4$, $T_1 = 1$. Then,

$$x(t) = 2.5x_0(t+1) - 0.5$$

$$FS\{x(t)\} = 2.5e^{jk\frac{2\pi}{T}1} FS\{x_0(t)\} - 0.5FS\{1\}$$

$$= \begin{cases} 2.5e^{jk\frac{\pi}{2}} \dfrac{\sin k\pi/2}{k\pi} & for\ k \neq 0 \\ 3/4 & for\ k = 0 \end{cases}$$

Note: FS{1} = 1 for k = 0, and 0 otherwise.

Also, the expression for $k \neq 0$ can be further simplified to $2.5j\dfrac{\sin^2 k\pi/2}{k\pi}$

**Problem 2:** Let *X(jw)* be the Fourier transform of $x(t) = e^{-2|t|}$. Find *X(j0)* and $\int_{-\infty}^{\infty} X(jw)dw$.

Using the definition of the Fourier transform:

$$X(j0) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt\Big|_{w=0} = \int_{-\infty}^{\infty} x(t)dt = 2\int_0^{\infty} e^{-2t}dt = 1$$

$$\int_{-\infty}^{\infty} X(jw)dw = 2\pi\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw\Big|_{t=0}\right] = 2\pi x(0) = 2\pi$$

Note: In this case, it is also possible to compute the Fourier transform of *x(t)*:

$$F\{x(t)\} = F\{e^{-2t}u(t)\} + F\{e^{2t}u(-t)\}$$

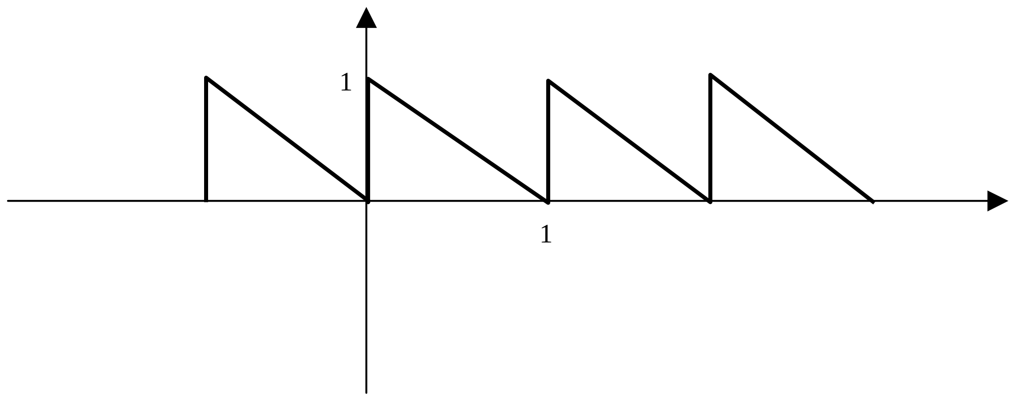
$$= \frac{1}{2+jw} + FR\{e^{-2t}u(+t)\} = \frac{1}{2+jw} + RF\{e^{-2t}u(t)\} = \frac{1}{2+jw} + R\left\{\frac{1}{2+jw}\right\} = \frac{1}{2+jw} + \frac{1}{2-jw} = \frac{4}{4+w^2}$$

**NAME:___SOLUTIONS____**

**EEE 203** **TEST 3**

**10/11/06, 30', closed books and notes, transform tables allowed**

**Problem 1:** Let *x(t)* be the periodic "sawtooth wave" signal shown in the figure below. Compute the coefficients $a_k$ of the Fourier series expansion of *x(t).*




$$T = 1, \quad w_0 = \frac{2\pi}{T} = 2\pi$$

$$\text{Direct computation : } a_0 = \frac{1}{T}\int_{<T>} x(t)dt = \tfrac{1}{2}$$

$$FS\{x\} = \frac{1}{jkw_0} FS\left\{\frac{dx}{dt}\right\} \quad (k \neq 0)$$

$$FS\left\{\frac{dx}{dt}\right\} = FS\left\{-1(t) + \sum_n \delta(t-n)\right\} = FS\{-1(t)\} + FS\left\{\sum_n \delta(t-n)\right\} = \begin{cases} -1+1 & k=0 \\ 0+1 & k \neq 0 \end{cases}$$

$$\Rightarrow FS\{x\} = \begin{cases} \tfrac{1}{2} & k=0 \\ \dfrac{1}{j2\pi k} & k \neq 0 \end{cases}$$

Note: Parseval's Theorem states $\frac{1}{T}\int_{<T>} |x|^2 = \sum_k |a_k|^2$. Applied to our case:

$\frac{1}{T}\int_{<T>} |x|^2 = \frac{1}{4} + \frac{2}{4\pi^2}\sum_1^{\infty} \tfrac{1}{k^2} = \int_0^1 t dt$. Complete the computation to derive the well-known formula for the series $1/k^2$.

**NAME____SOLUTIONS_____**

**EEE 203** **HW 4** **10/18/06**

**Problem 1:**

Consider the filter with impulse response $h(t) = e^{-2t}u(t)$.

1. Find the transfer function
2. Sketch the Bode Plot
3. Find the Fourier transform of the output when $x(t) = e^{-t}u(t)$
4. Find the Fourier transform of the output when $x(t) = \sin(2t)u(t)$
5. Find the output when $x(t) = e^{-t}u(t)$ (a) using convolution, and (b) taking the inverse Fourier transform of your answer to part 3.
6. Find the output when $x(t) = \sin(2t)u(t)$ (a) using convolution, and (b) taking the inverse Fourier transform of your answer to part 4.

1. $H(jw) = \dfrac{1}{jw+2}$

2. $|H(jw)| = \dfrac{1}{\sqrt{w^2+2^2}}, \quad \angle H(jw) = \tan^{-1}\left(\dfrac{w}{2}\right)$ (see attached plots)

3. $F\{x\} = \dfrac{1}{jw+1}, \quad Y(jw) = H(jw)X(jw) = \dfrac{1}{jw+2}\dfrac{1}{jw+1}$

4. $F\{x\} = \dfrac{1}{2\pi}F\{\sin 2t\} * F\{u(t)\} = \dfrac{1}{2j}[\delta(w-2)-\delta(w+2)] * \left[\dfrac{1}{jw}+\pi\delta(w)\right]$

$$= \frac{1}{2}\left[\frac{1}{w+2}-\frac{1}{w-2}\right] + \frac{\pi}{2j}[\delta(w-2)-\delta(w+2)]$$

$$Y(jw) = H(jw)X(jw) = \frac{1}{2}\left[\frac{1}{w+2}\frac{1}{jw+2}-\frac{1}{w-2}\frac{1}{jw+2}\right] + \frac{\pi}{2j}\left[\frac{1}{j2+2}\delta(w-2)-\frac{1}{-j2+2}\delta(w+2)\right]$$

5. $y(t) = (h*x)(t) = \int_{-\infty}^{\infty} e^{-2(t-\tau)}u(t-\tau)e^{-\tau}u(\tau)d\tau = e^{-2t}u(t)\int_0^t e^{\tau}d\tau = [e^{-t}-e^{-2t}]u(t)$

$$= F^{-1}\{Y(jw)\} = F^{-1}\left\{\frac{1}{jw+2}\frac{1}{jw+1}\right\} = F^{-1}\left\{\frac{-1}{jw+2}+\frac{1}{jw+1}\right\} = -e^{-2t}u(t)+e^{t}u(t)$$

6. $y(t) = (h*x)(t) = \int_{-\infty}^{\infty} e^{-2(t-\tau)} u(t-\tau)\sin(2\tau)u(\tau)d\tau = e^{-2t}u(t)\int_0^t e^{2\tau}\left[\frac{e^{j2\tau} - e^{-j2\tau}}{2j}\right]d\tau$

$$= \frac{e^{-2t}u(t)}{2j}\left[\frac{e^{(2+2j)t} - 1}{2+2j} - \frac{e^{(2-2j)t} - 1}{2-2j}\right] = \frac{u(t)}{2j}\left[\frac{e^{2jt} - e^{-2t}}{2+2j} - \frac{e^{-2jt} - e^{-2t}}{2-2j}\right]$$

$$= \Im m\left[\frac{e^{2jt} - e^{-2t}}{2+2j}\right]u(t) \qquad \Im m = \text{Imaginary part}$$

$$= \tfrac{1}{4}\left[e^{-2t} - \cos 2t + \sin 2t\right]u(t)$$

$$= F^{-1}\{Y(jw)\} = F^{-1}\left\{\frac{1}{2}\left[\frac{1}{w+2}\frac{1}{jw+2} - \frac{1}{w-2}\frac{1}{jw+2}\right]\right\} + F^{-1}\left\{\frac{\pi}{2j}\left[\frac{1}{j2+2}\delta(w-2) + \frac{1}{j2-2}\delta(w+2)\right]\right\}$$

$$= \frac{1}{2}F^{-1}\left\{\left[\frac{j}{jw+2j}\frac{1}{jw+2} - \frac{j}{jw-2j}\frac{1}{jw+2}\right] + \left[\frac{\pi}{-2+2j}\delta(w-2) - \frac{\pi}{2+2j}\delta(w+2)\right]\right\}$$

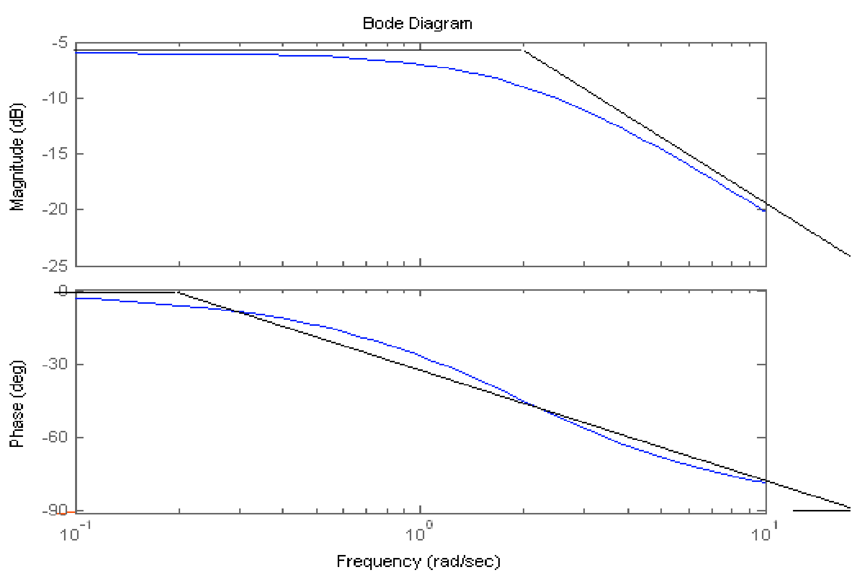
$$= \frac{1}{2}F^{-1}\left\{\left[\frac{j(2-2j)^{-1}}{jw+2j} + \frac{j(2j-2)^{-1}}{jw+2} - \frac{j(2+2j)^{-1}}{jw-2j} + \frac{j(2+2j)^{-1}}{jw+2}\right] + \frac{\pi}{2j-2}\delta(w-2) - \frac{\pi}{2j+2}\delta(w+2)\right\}$$

$$= \frac{1}{2}F^{-1}\left\{\frac{1}{-2j-2}\left[\frac{1}{j(w+2)} + \pi\delta(w+2)\right] + \frac{1}{2j-2}\left[\frac{1}{j(w-2)} + \pi\delta(w-2)\right] + \frac{(2+2j)^{-1} + (2-2j)^{-1}}{jw+2}\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{-2j-2}e^{-j2t}u(t) + \frac{1}{2j-2}e^{j2t}u(t) + 2\Re e(2+2j)^{-1}e^{-2t}u(t)\right\}$$

$$= \Re e\left\{\frac{e^{j2t}}{2j-2} + \frac{e^{-2t}}{2j+2}\right\}u(t) \qquad \Re e = \text{Real part}$$

$$= \tfrac{1}{4}\left[e^{-2t} - \cos 2t + \sin 2t\right]u(t)$$



**NAME___SOLUTIONS______**

**EEE 203** **Test 4**

CLOSED BOOK & NOTES. TRANSFORM TABLES ALLOWED. 30'

**Problem 1:**

Consider the filter with impulse response $h(t) = 2e^{-t}u(t)$.

1.(1pt) Find the transfer function

2.(2pt) Find the Fourier transform of the output *Y(jω)* when $x(t) = 3\sin t$

3.(2pt) Find the output *y(t)*.

1. $H(j\omega) = \dfrac{2}{j\omega + 1}$

2.
$$\begin{cases} X(j\omega) = \dfrac{3\pi}{j}[\delta(\omega-1) - \delta(\omega+1)] \Rightarrow Y(j\omega) = H(j\omega)X(j\omega) = \\ Y(j\omega) = \dfrac{2}{j\omega+1}\dfrac{3\pi}{j}[\delta(\omega-1) - \delta(\omega+1)] = \dfrac{2}{j1+1}\dfrac{3\pi}{j}\delta(\omega-1) - \dfrac{2}{j(-1)+1}\dfrac{3\pi}{j}\delta(\omega+1) \\ \quad = \dfrac{6\pi}{j-1}\delta(\omega-1) + \dfrac{6\pi}{-j-1}\delta(\omega+1) \end{cases}$$

3.
$$\begin{cases} y(t) = F^{-1}\{Y(j\omega)\} = 2\Re e F^{-1}\left\{\dfrac{6\pi}{j-1}\delta(\omega-1)\right\} = \Re e\left\{\dfrac{12\pi}{j-1}\dfrac{e^{jt}}{2\pi}\right\} = \Re e\left\{6\left|\dfrac{1}{j-1}\right| e^{jt + j\angle\frac{1}{j-1}}\right\} \\ \quad = 6\left|\dfrac{1}{\sqrt{2}}\right|\cos\left(t + \tan^{-1}\dfrac{1}{1} - 180^o\right) = 3\sqrt{2}\sin\left(t - 45^o\right) \\ \text{ALT.}: y(t) = |H(j\omega)|_{\omega=1}|(3)\sin[t + \angle H(j\omega)|_{\omega=1}] = 3\sqrt{2}\sin(t - \tan^{-1} 1/1) = 3\sqrt{2}\sin(t - 45^o) \end{cases}$$

**Problem 2:**

Consider the filter with impulse response $h(t) = \dfrac{\sin 2t}{t}$.

1.(1pt) Find the transfer function

2.(2pt) Find the Fourier transform of the output *Y(jω)* when $x(t) = \sin 3t$

3.(2pt) Find the output *y(t)*.

1. $H(j\omega) = \pi[u(\omega+2) - u(\omega-2)]$

2.
$$\begin{cases} X(j\omega) = \dfrac{\pi}{j}[\delta(\omega-3) - \delta(\omega+3)] \Rightarrow Y(j\omega) = H(j\omega)X(j\omega) = \\ Y(j\omega) = \pi[u(\omega+2) - u(\omega-2)]\dfrac{\pi}{j}[\delta(\omega-3) - \delta(\omega+3)] = 0 \end{cases}$$

3. $y(t) = F^{-1}\{Y(j\omega)\} = F^{-1}\{0\} = 0$

# EEE 203 HW 5 NAME:___SOLUTIONS____

**Problem 1:**
Find the largest sampling interval $T_s$ to allow perfect reconstruction of the signals:
(NOTE: h*x denotes convolution of h and x)

1. $\frac{\sin 2t}{t} * \frac{\sin t}{t}$

$$F\left\{\frac{\sin 2t}{t} * \frac{\sin t}{t}\right\} = F\left\{\frac{\sin 2t}{t}\right\} F\left\{\frac{\sin t}{t}\right\} = \{\pi u(\omega+2) - \pi u(\omega-2)\}\{\pi u(\omega+1) - \pi u(\omega-1)\}$$
$$= \pi^2 \{u(\omega+1) - u(\omega-1)\}.$$

$$\Rightarrow \text{max. freq.} = 1 \text{ rad/s, Nyquist freq.} = 2 \text{ rad/s, min. sampling interval} = T_s = \frac{2\pi}{2\omega_{\max}} = \frac{2\pi}{\omega_{Nyq}} = \pi$$

2. $\frac{\sin^2 t}{t^2}$

$$F\left\{\frac{\sin^2 t}{t^2}\right\} = \frac{1}{2\pi} F\left\{\frac{\sin t}{t}\right\} * F\left\{\frac{\sin t}{t}\right\} = \frac{1}{2\pi}\{\pi u(\omega+1) - \pi u(\omega-1)\} * \{\pi u(\omega+1) - \pi u(\omega-1)\}$$
$$= \frac{\pi}{2}\{r(\omega+2) - 2r(\omega) + r(\omega-2)\}.$$

$$\Rightarrow \text{max. freq.} = 2 \text{ rad/s, Nyquist freq.} = 4 \text{ rad/s, min. sampling interval} = T_s = \frac{2\pi}{2\omega_{\max}} = \frac{2\pi}{\omega_{Nyq}} = \frac{\pi}{2}$$

3. $\frac{\sin t}{t} * \sin 2t$

$$F\left\{\frac{\sin t}{t} * \sin 2t\right\} = F\left\{\frac{\sin t}{t}\right\} F\{\sin 2t\} = \{\pi u(\omega+1) - \pi u(\omega-1)\}\left\{\frac{\pi}{j}\delta(\omega-2) - \frac{\pi}{j}\delta(\omega+2)\right\}$$
$$= 0.$$

$$\Rightarrow \text{max. freq.} = 0 \text{ rad/s, Nyquist freq.} = 0 \text{ rad/s, min. sampling interval} = T_s = \frac{2\pi}{2\omega_{\max}} = \frac{2\pi}{\omega_{Nyq}} = \infty$$

i.e., the zero or any constant function can be sampled with arbitrarily large sampling intervals

**Problem 2:**
For a sampling process with rate 1ms, what is the cutoff frequency of the ideal low-pass filter needed for reconstruction?

The filter cutoff frequency should be the same as the maximum allowed signal frequency:

$$T_s = 1ms \Rightarrow 2\omega_{\max} = \frac{2\pi}{T_s} \Rightarrow \omega_{\max} = 1000\pi \text{ rad/s} = 500 \text{ Hz}.$$

**EEE 203** **TEST 5** **NAME:___SOLUTIONS___**

CLOSED BOOK & NOTES. TRANSFORM TABLES ALLOWED. 30'

**Problem 1:**

Find the largest sampling interval $T_s$ to allow perfect reconstruction of the signals:
(NOTE: h*x denotes convolution of h and x)

1. $\frac{\sin 2t}{t} * \cos t$.

$$F\left\{\frac{\sin 2t}{t} * \cos t\right\} = F\left\{\frac{\sin 2t}{t}\right\} F\{\cos t\} = \{\pi u(\omega+2) - \pi u(\omega-2)\}\{\pi\delta(\omega-1) + \pi\delta(\omega+1)\}$$

$$= \pi^2\{\delta(\omega-1) + \delta(\omega+1)\}$$

$$\Rightarrow \text{max. freq.} = 1 \text{ rad/s, Nyquist freq.} = 2 \text{ rad/s, min. sampling interval} = T_s = \frac{\pi}{\omega_{\max}} = \frac{2\pi}{\omega_{Nyq}} = \pi$$

2. $\frac{\sin t}{t} \cos 2t$.

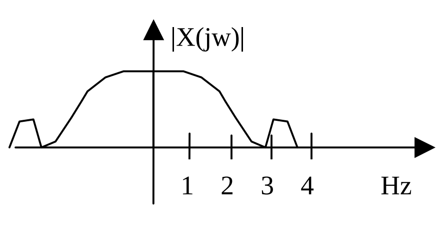
$$F\left\{\frac{\sin t}{t} \cos 2t\right\} = \frac{1}{2\pi} F\left\{\frac{\sin t}{t}\right\} * F\{\cos 2t\} = \frac{1}{2\pi}\{\pi u(\omega+1) - \pi u(\omega-1)\} * \{\pi\delta(\omega-2) + \pi\delta(\omega+2)\}$$

$$= \frac{\pi}{2}\{u(\omega-1) - u(\omega+3) + u(\omega+3) - u(\omega+1)\}$$

$$\Rightarrow \text{max. freq.} = 3 \text{ rad/s, Nyquist freq.} = 6 \text{ rad/s, min. sampling interval} = T_s = \frac{\pi}{\omega_{\max}} = \frac{2\pi}{\omega_{Nyq}} = \frac{\pi}{3}$$

**Problem 2:**

The frequency spectrum of a signal is shown in the figure below. Determine the sampling rate and the cutoff frequency of the ideal low-pass filter needed for reconstruction.




The maximum frequency in the signal is approximately 4 Hz, or, 4(2π) rad/s. Therefore maximum sampling time, to allow reconstruction, is π/8 π = 1/8 s. (=$1/2f_{max}$)

The ideal low-pass filter cutoff is the same as the maximum frequency in the signal, $w_c$ = 8 π rad/s = 4 Hz.

**EEE 203** **HW 6** **NAME:__________________**

**Problem 1:**
Consider the causal filter described by the difference equation

$$y[n] = \frac{1}{4} y[n-1] + \frac{1}{3} x[n-1]$$

1. Determine the transfer function
2. Compute the response of the filter to $x[n] = u[n]$
3. Compute the steady state response to $x[n] = u[n]$
4. Compute the steady state response to $x[n] = \sin(n\pi/6)\, u[n-1]$

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**Problem 2:**
Consider the continuous time causal filter with transfer function

$$H(s) = \frac{1}{(s+1)(s-1)}$$

Compute the response of the filter to $x[t] = u[t]$

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**Problem 3:**
Consider the discrete time stable filter with transfer function

$$H(z) = \frac{z-1}{(z-0.9)(z-1.1)}$$

Compute the response of the filter to $x[n] = u[n]$

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**EEE 203** **HW 6** **NAME:_____SOLUTIONS_**

**Problem 1:** Consider the causal filter described by the difference equation

$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{3}x[n-1]$$

1. Determine the transfer function
2. Compute the response of the filter to $x[n] = u[n]$
3. Compute the steady state response to $x[n] = u[n]$
4. Compute the steady state response to $x[n] = \sin(n\pi/6)\, u[n\text{-}1]$

Note: For a stable system if x converges to a periodic signal x_s, then y converges to a periodic signal that is the system response to the x_s. This can be computed using Fourier theory. A useful simplification is:

For Continuous Time:
$$x(t) = e^{j\omega_o t} \Rightarrow y(t) = H(j\omega_o)e^{j\omega_o t}$$
$$x(t) = \cos(\omega_o t) \Rightarrow y(t) = |H(j\omega_o)|\cos(\omega_o t + \angle H(j\omega_o))$$

For Discrete Time:
$$x(n) = e^{j\Omega_o n} \Rightarrow y(n) = H(e^{j\Omega_o})e^{j\Omega_o n}$$
$$x(n) = \cos(\Omega_o n) \Rightarrow y(n) = |H(e^{j\Omega_o})|\cos(\Omega_o n + \angle H(e^{j\Omega_o}))$$

where, $H(s), H(z)$ are the continuous and discrete system transfer functions, respectively.

1.

$$H(z) = \frac{\frac{1}{3}z^{-1}}{1 - \frac{1}{4}z^{-1}} = \frac{\frac{1}{3}}{z - \frac{1}{4}}$$

2. $y(z) = H(z)\,\frac{z}{z-1} = \frac{\frac{1}{3}z}{\left(z-\frac{1}{4}\right)(z-1)} = \frac{-\frac{4}{9}z}{\left(z-\frac{1}{4}\right)} + \frac{\frac{4}{9}z}{(z-1)} \Rightarrow y(n) = Z^{-1}\left\{\frac{-\frac{4}{9}z}{\left(z-\frac{1}{4}\right)} + \frac{\frac{4}{9}z}{(z-1)}\right\} =$

$$y(n) = -\frac{4}{9}\left(\frac{1}{4}\right)^n u(n) + \frac{4}{9}u(n) \Rightarrow \begin{bmatrix} n \\ y(n) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0.333 & 0.417 & 0.438 \end{bmatrix}$$

3. The filter is stable since the pole (1/4) has magnitude less than one. Hence, the steady-state response is well-defined. For the constant steady-state, $y(n) = H(z=1)\cos 0n = \frac{4}{9}$

4. $y_{ss}(n) = \left|H\left(e^{j\Omega}\right)\right| \sin\left(\Omega n + \angle H\left(e^{j\Omega}\right)\right),\ \Omega = \frac{\pi}{6}$

Then, $y_{ss}(n) = \dfrac{\frac{1}{3}}{\sqrt{\left\{\left[\cos\left(\frac{\pi}{6}\right) - \frac{1}{4}\right]^2 + \left[\sin\left(\frac{\pi}{6}\right)\right]^2\right\}}} \sin\left(\frac{\pi}{6}n - \operatorname{atan}\frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right) - \frac{1}{4}}\right) = 0.42\sin\left(\frac{\pi}{6}n - 39^o\right)$

**Problem 2:** Consider the continuous time causal filter with transfer function

$$H(s) = \frac{1}{(s+1)(s-1)}$$

Compute the response of the filter to $x[t] = u[t]$

$$y(s) = H(s)\frac{1}{s} = \frac{1}{s(s+1)(s-1)}$$
$$y(t) = L^{-1}\left\{\frac{-1}{s} + \frac{1/2}{(s+1)} + \frac{1/2}{(s-1)}\right\}$$
$$y(t) = (-1)u(t) + 1/2e^{-t}u(t) + 1/2e^{t}u(t)$$

**Problem 3:** Consider the discrete time causal filter with transfer function

$$H(z) = \frac{z-1}{(z-0.9)(z-1.1)}$$

Compute the response of the filter to $x[n] = u[n]$

$$y(z) = H(z)\frac{z}{z-1} = \frac{z}{(z-0.9)(z-1.1)}$$

$$y(n) = Z^{-1}\left\{z\left(\frac{-\frac{1}{0.2}}{(z-0.9)} + \frac{\frac{1}{0.2}}{(z-1.1)}\right)\right\} = Z^{-1}\left\{\frac{-5z}{(z-0.9)} + \frac{5z}{(z-1.1)}\right\}$$

$$y(k) = (-5)0.9^k u(k) + (5)1.1^k u(k) \Rightarrow \begin{bmatrix} k \\ y(k) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3.01 & 4.04 & 5.1 \end{bmatrix}$$

**EEE 203** **TEST 6** **NAME:___SOLUTIONS___**

CLOSED BOOK & NOTES. TRANSFORM TABLES ALLOWED. 60'

**Problem 1:**

For the continuous time system with transfer function

$$H(s) = \frac{200(s-1)}{(s+10)(s+20)}$$

1. Find the ROC, assuming that the system is stable.
2. Find the steady-state response to *x(t) = u(t)*.
3. Find the amplitude of the steady-state output for a sinusoid input of frequency 1 rad/s (*x(t)* = sin(*t*)).

1. The poles are at –10, –20. Since the system is stable the ROC includes the jw-axis and extends to the nearest pole (left and right). So, ROC = Re s > -10

2. $y_{ss} = H[x_{ss}] = \left.\frac{200(s-1)}{(s+10)(s+20)}\right|_{s=j0} 1 = \frac{200(0-1)}{(0+10)(0+20)} = 1$

3. $y_{ss}(t) = |H(j1)|\sin(t + \angle H(j1)) \Rightarrow Ampl. = \left|\left.\frac{200(s-1)}{(s+10)(s+20)}\right|_{s=j1}\right| = \frac{200\sqrt{2}}{\sqrt{101}\sqrt{401}} = 1.405$

**Problem 2:**

For the discrete time system with transfer function

$$H(z) = \frac{1}{(z+0.5)(z-0.5)}$$

1. Find the ROC, assuming that the system is stable.
2. Find the ROC, assuming that the system is causal
3. For the stable *H(z)*, compute the response to a step *x[n] = u[n]*.

1. The poles are at –0.5, 0.5. Since the system is stable the ROC includes the unit circle and extends to the nearest pole. So, ROC = {z : |z| >0.5}.
2. The ROC of a causal transfer function extends from the largest (in magnitude) pole, to infinity. So, ROC = {z : |z| >0.5}.

$$Z\{u[n]\} = \frac{z}{z-1}$$

2. $Y(z) = H(z)X(z) = \frac{1}{(z+0.5)(z-0.5)}\frac{z}{z-1} = \frac{{}^{-0.5}\!/_{(-1)(-1.5)}}{z+0.5} + \frac{{}^{0.5}\!/_{(1)(-0.5)}}{z-0.5} + \frac{{}^{1}\!/_{(1.5)(0.5)}}{z-1}$

$$y[n] = Z^{-1}\{Y(z)\} = \tfrac{-1}{3}(-0.5)^{n-1}u[n-1] + (-1)(0.5)^{n-1}u[n-1] + \tfrac{4}{3}u[n-1]$$