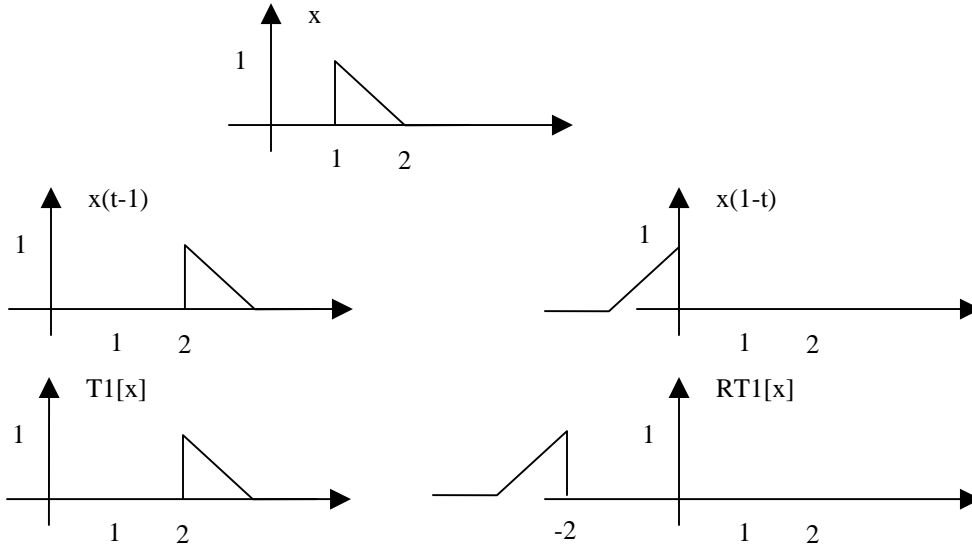


EEE 203

HW 1

Problem 1.

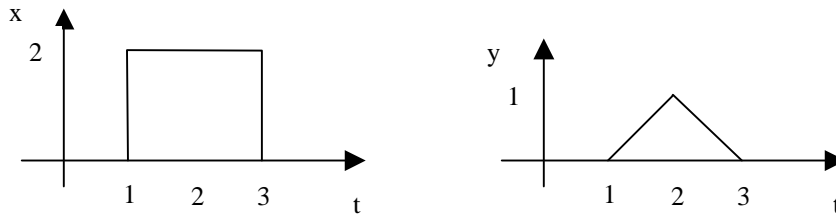
Consider the signal $x(t)$ whose graph is shown below. Sketch the following signals: $x(t-1)$, $x(1-t)$, $RT_1[x]$, where R denotes the reflection operation and T_{t_0} denotes shift delay operation by t_0 .



Problem 2.

Describe the following signals in terms of elementary functions (δ , u , r , ...) and compute

$$\int_{-\infty}^{\infty} x(t)\delta(t-1)dt \text{ and } \int_{-\infty}^{\infty} y(t)\delta(t-1)dt.$$



$$x(t) = 2u(t-1) - 2u(t-3)$$

$$y(t) = r(t-1) - 2r(t-2) + r(t-3)$$

$$\int_{-\infty}^{\infty} x(t)\delta(t-1)dt = \frac{x(1^+) + x(1^-)}{2} = 1$$

$$\int_{-\infty}^{\infty} y(t)\delta(t-1)dt = \frac{y(1^+) + y(1^-)}{2} = 0$$

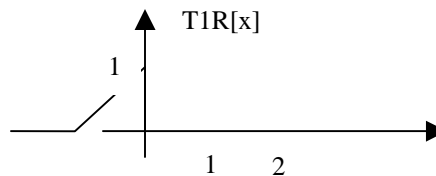
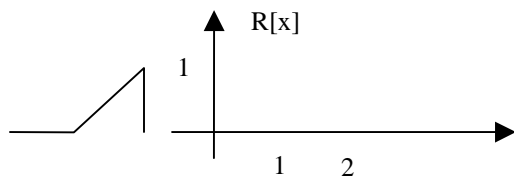
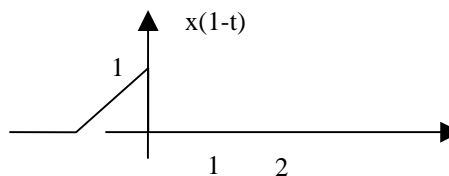
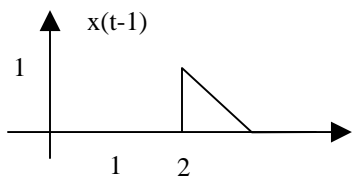
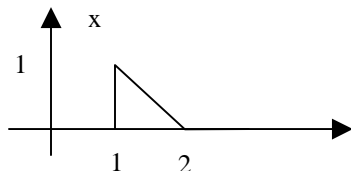
NAME: _____

EEE 203

TEST 1

Problem 1. (4pts)

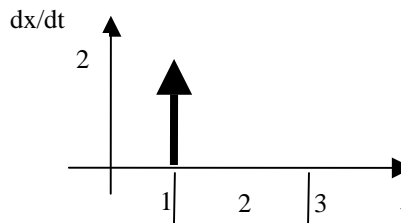
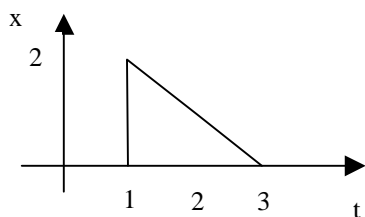
Consider the signal $x(t)$ whose graph is shown below. Sketch the following signals: $x(t-1)$, $x(1-t)$, $T_1R[x]$, where R denotes the reflection operation and T_{t_0} denotes shift delay operation by t_0 .



Problem 2. (6pts)

Describe the following signal in terms of elementary functions (δ , u , r , ...) and compute

$\frac{dx(t)}{dt}$ and $\int_{-\infty}^{\infty} x(t+1)\delta(t-1)dt$.



$$x(t) = 2u(t-1) - r(t-1) + r(t-3)$$

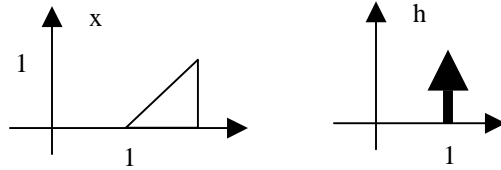
$$\frac{dx(t)}{dt} = 2\delta(t-1) - u(t-1) + u(t-3)$$

$$\begin{aligned} \int_{-\infty}^{\infty} x(t+1)\delta(t-1)dt &= \int_{-\infty}^{\infty} x(1+1)\delta(t-1)dt \\ &= 1 \int_{-\infty}^{\infty} \delta(t-1)dt = 1 \end{aligned}$$

EEE 203

HW 2

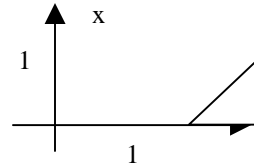
Problem 1. Compute the convolution $h*x$ when $x(t) = r(t)-r(t-1)-u(t-1)$, $h(t)=\delta(t-1)$.



($u(t)$ is the unit step)

Unfortunately, the $x(t)$ shown is a delayed version of the expression: $x(t) = r(t-1)-r(t-2)-u(t-2)$.

$(h*x)(t) = [(\delta(t-1))*x](t) = x(t-1) = r(t-2)-r(t-3)-u(t-3)$



Problem 2.

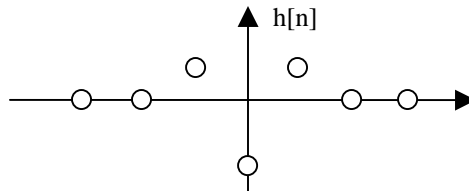
Consider the filters:

A. $y[n] = x[n + 1] - 2x[n] + x[n - 1]$

B. $y(t) = \int_{-\infty}^{t+1} e^{t-\tau} x(\tau) d\tau$

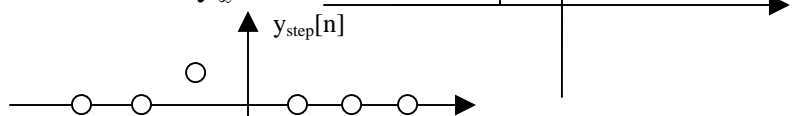
1. Find and graph their impulse responses.

A. $h[n] = \delta[n + 1] - 2\delta[n] + \delta[n - 1]$



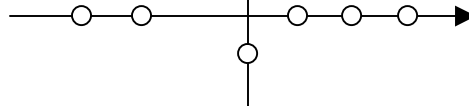
B. $y(t) = \int_{-\infty}^{t+1} e^{t-\tau} x(\tau) d\tau = \int_{-\infty}^{\infty} e^{t-\tau} u(t+1-\tau)x(\tau) d\tau = \int_{-\infty}^{\infty} h(t-\tau)x(\tau) d\tau$

$h(t) = e^t u(t+1)$

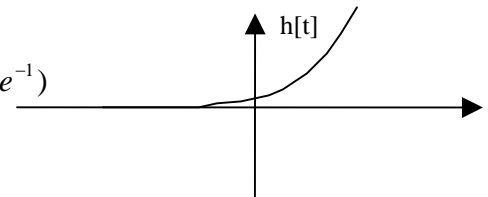


2. Find and graph their step responses.

A. $y_{step}[n] = u[n + 1] - 2u[n] + u[n - 1]$



B. $y_{step}(t) = \int_{-\infty}^{t+1} e^{t-\tau} u(\tau) d\tau = u(t+1) \int_0^{t+1} e^{t-\tau} d\tau = u(t+1)(e^t - e^{-1})$



3. Which filters are causal? (Justify)

A. Not causal, $h(-1)$ is not zero.

B. Not causal, $h(t)$ is nonzero in $[-1,0)$.

4. Which filters are stable? (Justify)

A. Stable since it is FIR. $h[n]$ is absolutely summable. ($\sum |h[n]| = 1+2+1 = 4 < \infty$.)

B. Unstable since $h(t)$ diverging implies that $\int_{-\infty}^{\infty} |h(t)| dt$ diverges.

EEE 203

TEST 2

Wed. 9/27/06

30', closed books&notes

Problem 1. Compute the convolution $h*x$ when $x(t) = u(t) - u(t-1)$, $h(t)=u(t)$.

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau = \int_{-\infty}^{\infty} u(t-\tau)u(\tau)d\tau - \int_{-\infty}^{\infty} u(t-\tau)u(\tau-1)d\tau \\ &= u(t)\int_0^t 1d\tau - u(t-1)\int_1^t 1d\tau = tu(t) - (t-1)u(t-1) = r(t) - r(t-1) \end{aligned}$$

Problem 2.

Consider the filters:

A. $y[n] = x[n] - x[n-1]$

B. $y(t) = \int_{-\infty}^{t-1} x(\tau)d\tau$

1. Find and graph their impulse responses.

A. $h(n) = \delta(n) - \delta(n-1)$

B. $y(t) = \int_{-\infty}^{\infty} u(t-1-\tau)x(\tau)d\tau \Rightarrow h(t) = u(t-1)$

2. Which filters are causal? (Justify)

A. Causal because $h(n) = 0$, for all $n < 0$.

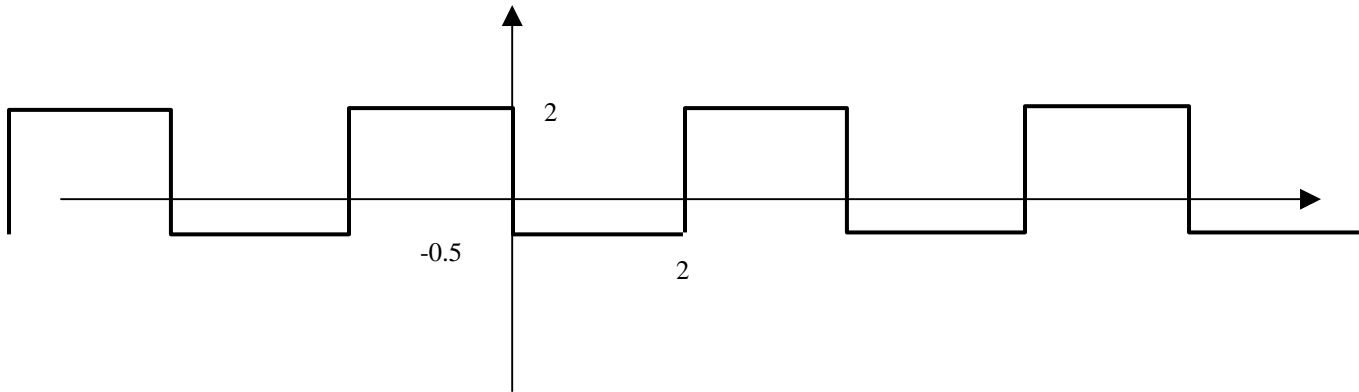
B. Causal because $h(t) = 0$, for all $t < 0$.

3. Which filters are stable? (Justify)

A. Stable because $\sum |h(n)| = 2 < \infty$

B. Unstable because $\int |h(t)| dt = \lim_{T \rightarrow \infty} \int_1^T 1dt$, which diverges.

Problem 1: Let $x(t)$ be the periodic signal shown in the figure below (square wave with offset). Compute the coefficients a_k of the Fourier series expansion of $x(t)$.



Let $x_0(t)$ be the standard square wave (in the Tables), with $T = 4$, $T_1 = 1$. Then,
 $x(t) = 2.5x_0(t + 1) - 0.5$

$$FS\{x(t)\} = 2.5e^{jk\frac{2\pi}{T}1} FS\{x_0(t)\} - 0.5FS\{1\}$$

$$= \begin{cases} 2.5e^{jk\frac{\pi}{2}} \frac{\sin k\pi/2}{k\pi} & \text{for } k \neq 0 \\ 3/4 & \text{for } k = 0 \end{cases}$$

Note: $FS\{1\} = 1$ for $k = 0$, and 0 otherwise.

Also, the expression for $k \neq 0$ can be further simplified to $2.5j \frac{\sin^2 k\pi/2}{k\pi}$

Problem 2: Let $X(j\omega)$ be the Fourier transform of $x(t) = e^{-2|t|}$. Find $X(j0)$ and $\int_{-\infty}^{\infty} X(j\omega)d\omega$.

Using the definition of the Fourier transform:

$$X(j0) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \Big|_{\omega=0} = \int_{-\infty}^{\infty} x(t) dt = 2 \int_0^{\infty} e^{-2t} dt = 1$$

$$\int_{-\infty}^{\infty} X(j\omega)d\omega = 2\pi \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)e^{j\omega t} d\omega \Big|_{t=0} \right] = 2\pi x(0) = 2\pi$$

Note: In this case, it is also possible to compute the Fourier transform of $x(t)$:

$$F\{x(t)\} = F\{e^{-2t}u(t)\} + F\{e^{2t}u(-t)\}$$

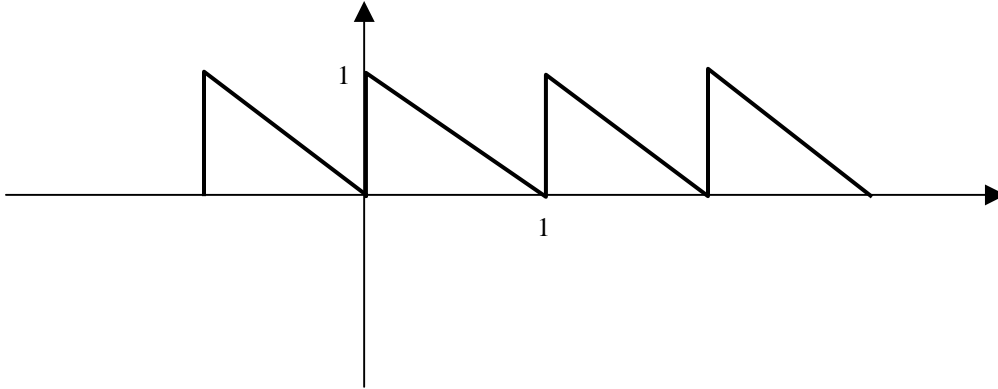
$$= \frac{1}{2 + j\omega} + FR\{e^{-2t}u(+t)\} = \frac{1}{2 + j\omega} + RF\{e^{-2t}u(t)\} = \frac{1}{2 + j\omega} + R\left\{ \frac{1}{2 + j\omega} \right\} = \frac{1}{2 + j\omega} + \frac{1}{2 - j\omega} = \frac{4}{4 + \omega^2}$$

EEE 203

TEST 3

10/11/06, 30', closed books and notes, transform tables allowed

Problem 1: Let $x(t)$ be the periodic “sawtooth wave” signal shown in the figure below. Compute the coefficients a_k of the Fourier series expansion of $x(t)$.



$$T = 1, \quad \omega_0 = \frac{2\pi}{T} = 2\pi$$

$$\text{Direct computation: } a_0 = \frac{1}{T} \int_{\langle T \rangle} x(t) dt = \frac{1}{2}$$

$$FS\{x\} = \frac{1}{jk\omega_0} FS\left\{\frac{dx}{dt}\right\} \quad (k \neq 0)$$

$$FS\left\{\frac{dx}{dt}\right\} = FS\left\{-1(t) + \sum_n \delta(t-n)\right\} = FS\{-1(t)\} + FS\left\{\sum_n \delta(t-n)\right\} = \begin{cases} -1+1 & k=0 \\ 0+1 & k \neq 0 \end{cases}$$

$$\Rightarrow FS\{x\} = \begin{cases} \frac{1}{2} & k=0 \\ \frac{1}{j2\pi k} & k \neq 0 \end{cases}$$

Note: Parseval's Theorem states $\frac{1}{T} \int_{\langle T \rangle} |x|^2 = \sum_k |a_k|^2$. Applied to our case:

$\frac{1}{T} \int_{\langle T \rangle} |x|^2 = \frac{1}{4} + \frac{2}{4\pi^2} \sum_1^{\infty} \frac{1}{k^2} = \int_0^1 t dt$. Complete the computation to derive the well-known formula for the series $1/k^2$.

Problem 1:

Consider the filter with impulse response $h(t) = e^{-2t}u(t)$.

1. Find the transfer function
2. Sketch the Bode Plot
3. Find the Fourier transform of the output when $x(t) = e^{-t}u(t)$
4. Find the Fourier transform of the output when $x(t) = \sin(2t)u(t)$
5. Find the output when $x(t) = e^{-t}u(t)$ (a) using convolution, and (b) taking the inverse Fourier transform of your answer to part 3.
6. Find the output when $x(t) = \sin(2t)u(t)$ (a) using convolution, and (b) taking the inverse Fourier transform of your answer to part 4.

$$1. H(j\omega) = \frac{1}{j\omega + 2}$$

$$2. |H(j\omega)| = \frac{1}{\sqrt{\omega^2 + 2^2}}, \quad \angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{2}\right) \quad (\text{see attached plots})$$

$$3. F\{x\} = \frac{1}{j\omega + 1}, \quad Y(j\omega) = H(j\omega)X(j\omega) = \frac{1}{j\omega + 2} \frac{1}{j\omega + 1}$$

$$4. F\{x\} = \frac{1}{2\pi} F\{\sin 2t\} * F\{u(t)\} = \frac{1}{2j} [\delta(\omega - 2) - \delta(\omega + 2)] * \left[\frac{1}{j\omega} + \pi\delta(\omega) \right]$$

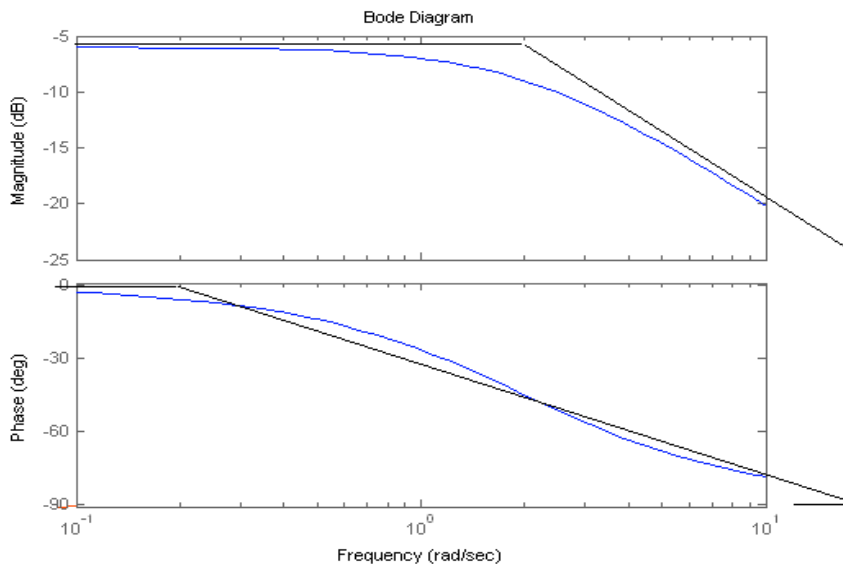
$$= \frac{1}{2} \left[\frac{1}{\omega + 2} - \frac{1}{\omega - 2} \right] + \frac{\pi}{2j} [\delta(\omega - 2) - \delta(\omega + 2)]$$

$$Y(j\omega) = H(j\omega)X(j\omega) = \frac{1}{2} \left[\frac{1}{\omega + 2} \frac{1}{j\omega + 2} - \frac{1}{\omega - 2} \frac{1}{j\omega + 2} \right] + \frac{\pi}{2j} \left[\frac{1}{j2 + 2} \delta(\omega - 2) - \frac{1}{-j2 + 2} \delta(\omega + 2) \right]$$

$$5. y(t) = (h * x)(t) = \int_{-\infty}^{\infty} e^{-2(t-\tau)} u(t-\tau) e^{-\tau} u(\tau) d\tau = e^{-2t} u(t) \int_0^t e^{\tau} d\tau = [e^{-t} - e^{-2t}] u(t)$$

$$= F^{-1}\{Y(j\omega)\} = F^{-1}\left\{ \frac{1}{j\omega + 2} \frac{1}{j\omega + 1} \right\} = F^{-1}\left\{ \frac{-1}{j\omega + 2} + \frac{1}{j\omega + 1} \right\} = -e^{-2t} u(t) + e^{-t} u(t)$$

$$\begin{aligned}
6. \quad y(t) &= (h * x)(t) = \int_{-\infty}^{\infty} e^{-2(t-\tau)} u(t-\tau) \sin(2\tau) u(\tau) d\tau = e^{-2t} u(t) \int_0^t e^{2\tau} \left[\frac{e^{j2\tau} - e^{-j2\tau}}{2j} \right] d\tau \\
&= \frac{e^{-2t} u(t)}{2j} \left[\frac{e^{(2+2j)t} - 1}{2+2j} - \frac{e^{(2-2j)t} - 1}{2-2j} \right] = \frac{u(t)}{2j} \left[\frac{e^{2jt} - e^{-2t}}{2+2j} - \frac{e^{-2jt} - e^{-2t}}{2-2j} \right] \\
&= \Im m \left[\frac{e^{2jt} - e^{-2t}}{2+2j} \right] u(t) \quad \Im m = \text{Imaginary part} \\
&= \frac{1}{4} \left[e^{-2t} - \cos 2t + \sin 2t \right] u(t) \\
&= F^{-1} \{ Y(j\omega) \} = F^{-1} \left\{ \frac{1}{2} \left[\frac{1}{\omega+2} \frac{1}{j\omega+2} - \frac{1}{\omega-2} \frac{1}{j\omega+2} \right] \right\} + F^{-1} \left\{ \frac{\pi}{2j} \left[\frac{1}{j2+2} \delta(\omega-2) + \frac{1}{j2-2} \delta(\omega+2) \right] \right\} \\
&= \frac{1}{2} F^{-1} \left\{ \left[\frac{j}{j\omega+2j} \frac{1}{j\omega+2} - \frac{j}{j\omega-2j} \frac{1}{j\omega+2} \right] + \left[\frac{\pi}{-2+2j} \delta(\omega-2) - \frac{\pi}{2+2j} \delta(\omega+2) \right] \right\} \\
&= \frac{1}{2} F^{-1} \left\{ \left[\frac{j(2-2j)^{-1}}{j\omega+2j} + \frac{j(2j-2)^{-1}}{j\omega+2} - \frac{j(2+2j)^{-1}}{j\omega-2j} + \frac{j(2+2j)^{-1}}{j\omega+2} \right] + \frac{\pi}{2j-2} \delta(\omega-2) - \frac{\pi}{2j+2} \delta(\omega+2) \right\} \\
&= \frac{1}{2} F^{-1} \left\{ \frac{1}{-2j-2} \left[\frac{1}{j(\omega+2)} + \pi \delta(\omega+2) \right] + \frac{1}{2j-2} \left[\frac{1}{j(\omega-2)} + \pi \delta(\omega-2) \right] + \frac{(2+2j)^{-1} + (2-2j)^{-1}}{j\omega+2} \right\} \\
&= \frac{1}{2} \left\{ \frac{1}{-2j-2} e^{-j2t} u(t) + \frac{1}{2j-2} e^{j2t} u(t) + 2\Re(2+2j)^{-1} e^{-2t} u(t) \right\} \\
&= \Re e \left\{ \frac{e^{j2t}}{2j-2} + \frac{e^{-2t}}{2j+2} \right\} u(t) \quad \Re e = \text{Real part} \\
&= \frac{1}{4} \left[e^{-2t} - \cos 2t + \sin 2t \right] u(t)
\end{aligned}$$



EEE 203

Test 4

CLOSED BOOK & NOTES. TRANSFORM TABLES ALLOWED. 30'

Problem 1:

Consider the filter with impulse response $h(t) = 2e^{-t}u(t)$.

- 1.(1pt) Find the transfer function
- 2.(2pt) Find the Fourier transform of the output $Y(j\omega)$ when $x(t) = 3 \sin t$
- 3.(2pt) Find the output $y(t)$.

1. $H(j\omega) = \frac{2}{j\omega + 1}$
2.
$$\left\{ \begin{aligned} X(j\omega) &= \frac{3\pi}{j} [\delta(\omega - 1) - \delta(\omega + 1)] \Rightarrow Y(j\omega) = H(j\omega)X(j\omega) = \\ Y(j\omega) &= \frac{2}{j\omega + 1} \frac{3\pi}{j} [\delta(\omega - 1) - \delta(\omega + 1)] = \frac{2}{j1 + 1} \frac{3\pi}{j} \delta(\omega - 1) - \frac{2}{j(-1) + 1} \frac{3\pi}{j} \delta(\omega + 1) \\ &= \frac{6\pi}{j-1} \delta(\omega - 1) + \frac{6\pi}{-j-1} \delta(\omega + 1) \end{aligned} \right.$$
3.
$$\left\{ \begin{aligned} y(t) &= F^{-1}\{Y(j\omega)\} = 2\Re e F^{-1}\left\{\frac{6\pi}{j-1} \delta(\omega - 1)\right\} = \Re e \left\{ \frac{12\pi}{j-1} \frac{e^{jt}}{2\pi} \right\} = \Re e \left\{ 6 \left| \frac{1}{j-1} \right| e^{jt + j\angle \frac{1}{j-1}} \right\} \\ &= 6 \left| \frac{1}{\sqrt{2}} \right| \cos\left(t + \tan^{-1} \frac{1}{1} - 180^\circ\right) = 3\sqrt{2} \sin(t - 45^\circ) \\ \text{ALT.: } y(t) &= |H(j\omega)|_{\omega=1} (3) \sin[t + \angle H(j\omega)|_{\omega=1}] = 3\sqrt{2} \sin(t - \tan^{-1} 1) = 3\sqrt{2} \sin(t - 45^\circ) \end{aligned} \right.$$

Problem 2:

Consider the filter with impulse response $h(t) = \frac{\sin 2t}{t}$.

- 1.(1pt) Find the transfer function
- 2.(2pt) Find the Fourier transform of the output $Y(j\omega)$ when $x(t) = \sin 3t$
- 3.(2pt) Find the output $y(t)$.

1. $H(j\omega) = \pi[u(\omega + 2) - u(\omega - 2)]$
2.
$$\left\{ \begin{aligned} X(j\omega) &= \frac{\pi}{j} [\delta(\omega - 3) - \delta(\omega + 3)] \Rightarrow Y(j\omega) = H(j\omega)X(j\omega) = \\ Y(j\omega) &= \pi[u(\omega + 2) - u(\omega - 2)] \frac{\pi}{j} [\delta(\omega - 3) - \delta(\omega + 3)] = 0 \end{aligned} \right.$$
3. $y(t) = F^{-1}\{Y(j\omega)\} = F^{-1}\{0\} = 0$

Problem 1:

Find the largest sampling interval T_s to allow perfect reconstruction of the signals:

(NOTE: $h*x$ denotes convolution of h and x)

$$1. \frac{\sin 2t}{t} * \frac{\sin t}{t}$$

$$F\left\{\frac{\sin 2t}{t} * \frac{\sin t}{t}\right\} = F\left\{\frac{\sin 2t}{t}\right\} F\left\{\frac{\sin t}{t}\right\} = \{\pi u(\omega + 2) - \pi u(\omega - 2)\} \{\pi u(\omega + 1) - \pi u(\omega - 1)\}$$

$$= \pi^2 \{u(\omega + 1) - u(\omega - 1)\}$$

$$\Rightarrow \text{max. freq.} = 1 \text{ rad/s, Nyquist freq.} = 2 \text{ rad/s, min. sampling interval} = T_s = \frac{2\pi}{2\omega_{\max}} = \frac{2\pi}{\omega_{\text{Nyq}}} = \pi$$

$$2. \frac{\sin^2 t}{t^2}$$

$$F\left\{\frac{\sin^2 t}{t^2}\right\} = \frac{1}{2\pi} F\left\{\frac{\sin t}{t}\right\} * F\left\{\frac{\sin t}{t}\right\} = \frac{1}{2\pi} \{\pi u(\omega + 1) - \pi u(\omega - 1)\} * \{\pi u(\omega + 1) - \pi u(\omega - 1)\}$$

$$= \frac{\pi}{2} \{r(\omega + 2) - 2r(\omega) + r(\omega - 2)\}$$

$$\Rightarrow \text{max. freq.} = 2 \text{ rad/s, Nyquist freq.} = 4 \text{ rad/s, min. sampling interval} = T_s = \frac{2\pi}{2\omega_{\max}} = \frac{2\pi}{\omega_{\text{Nyq}}} = \frac{\pi}{2}$$

$$3. \frac{\sin t}{t} * \sin 2t$$

$$F\left\{\frac{\sin t}{t} * \sin 2t\right\} = F\left\{\frac{\sin t}{t}\right\} F\{\sin 2t\} = \{\pi u(\omega + 1) - \pi u(\omega - 1)\} \left\{\frac{\pi}{j} \delta(\omega - 2) - \frac{\pi}{j} \delta(\omega + 2)\right\}$$

$$= 0$$

$$\Rightarrow \text{max. freq.} = 0 \text{ rad/s, Nyquist freq.} = 0 \text{ rad/s, min. sampling interval} = T_s = \frac{2\pi}{2\omega_{\max}} = \frac{2\pi}{\omega_{\text{Nyq}}} = \infty$$

i.e., the zero or any constant function can be sampled with arbitrarily large sampling intervals

Problem 2:

For a sampling process with rate 1ms, what is the cutoff frequency of the ideal low-pass filter needed for reconstruction?

The filter cutoff frequency should be the same as the maximum allowed signal frequency:

$$T_s = 1\text{ms} \Rightarrow 2\omega_{\max} = \frac{2\pi}{T_s} \Rightarrow \omega_{\max} = 1000\pi \text{ rad/s} = 500 \text{ Hz} .$$

Problem 1:

Find the largest sampling interval T_s to allow perfect reconstruction of the signals:

(NOTE: $h*x$ denotes convolution of h and x)

1. $\frac{\sin 2t}{t} * \cos t$.

$$F\left\{\frac{\sin 2t}{t} * \cos t\right\} = F\left\{\frac{\sin 2t}{t}\right\} F\{\cos t\} = \{\pi u(\omega + 2) - \pi u(\omega - 2)\} \{\pi \delta(\omega - 1) + \pi \delta(\omega + 1)\}$$

$$= \pi^2 \{\delta(\omega - 1) + \delta(\omega + 1)\}$$

$$\Rightarrow \text{max. freq.} = 1 \text{ rad/s, Nyquist freq.} = 2 \text{ rad/s, min. sampling interval} = T_s = \frac{\pi}{\omega_{\max}} = \frac{2\pi}{\omega_{\text{Nyq}}} = \pi$$

2. $\frac{\sin t}{t} \cos 2t$.

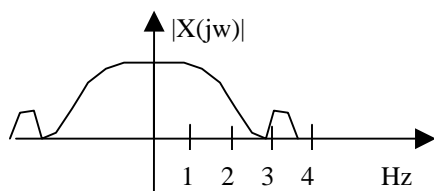
$$F\left\{\frac{\sin t}{t} \cos 2t\right\} = \frac{1}{2\pi} F\left\{\frac{\sin t}{t}\right\} * F\{\cos 2t\} = \frac{1}{2\pi} \{\pi u(\omega + 1) - \pi u(\omega - 1)\} * \{\pi \delta(\omega - 2) + \pi \delta(\omega + 2)\}$$

$$= \frac{\pi}{2} \{u(\omega - 1) - u(\omega + 3) + u(\omega + 3) - u(\omega + 1)\}$$

$$\Rightarrow \text{max. freq.} = 3 \text{ rad/s, Nyquist freq.} = 6 \text{ rad/s, min. sampling interval} = T_s = \frac{\pi}{\omega_{\max}} = \frac{2\pi}{\omega_{\text{Nyq}}} = \frac{\pi}{3}$$

Problem 2:

The frequency spectrum of a signal is shown in the figure below. Determine the sampling rate and the cutoff frequency of the ideal low-pass filter needed for reconstruction.



The maximum frequency in the signal is approximately 4 Hz, or, $4(2\pi)$ rad/s. Therefore maximum sampling time, to allow reconstruction, is $\pi/8$ s. ($=1/2f_{\max}$)

The ideal low-pass filter cutoff is the same as the maximum frequency in the signal, $\omega_c = 8\pi$ rad/s = 4 Hz.