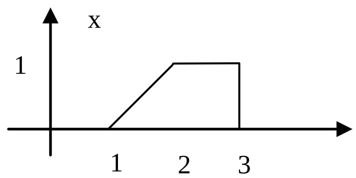
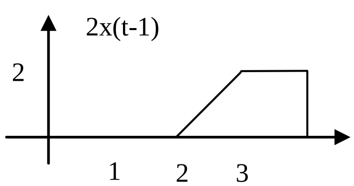
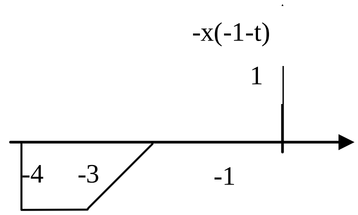
EEE 203, HW 1 **NAME:___SOLUTIONS____**

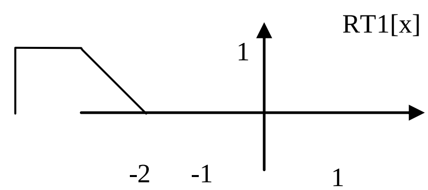
Problem 1.
Consider the signal *x(t)* whose graph is shown below. Sketch the following signals: *2x(t-1), -x(-1-t),* $RT_1[x]$, where R denotes the reflection operation and T_{t0} denotes shift delay operation by t0.



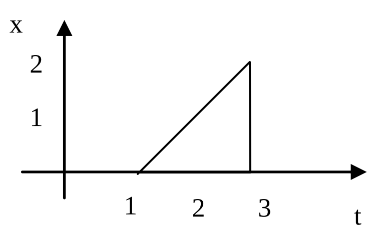
$x(t) = r(t-1) - r(t-2) - u(t-3)$
$2x(t-1) = 2r(t-2) - 2r(t-3) - 2u(t-4)$
$-x(-1-t) = -r(-1-t-1) + r(-1-t-2) + u(-1-t-3) = -r(-2-t) + r(-t-3) + u(-t-4)$
$T_1x(t) = r(t-1-1) - r(t-1-2) - u(t-1-3) = r(t-2) - r(t-3) - u(t-4)$
$RT_1x(t) = r(-t-2) - r(-t-3) - u(-t-4)$

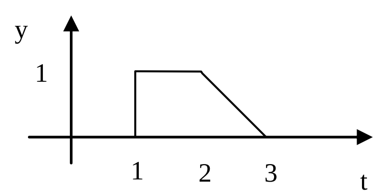






Problem 2.
Describe the following signals in terms of elementary functions (δ, *u* ,*r*, …) and compute $\int_{-\infty}^{\infty} x(t)\delta(t-3)dt$ and $\int_{-\infty}^{\infty} y(t)\delta(t+3)dt$.



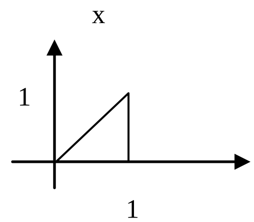


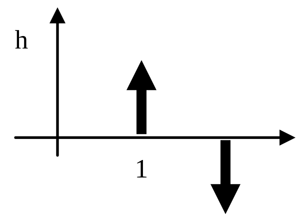
$x(t) = r(t-1) - r(t-3) - 2u(t-3) \Rightarrow \int x(t)\delta(t-3)dt = \frac{x(3^-)+x(3^+)}{2} = 1$

$y(t) = u(t-1) - r(t-2) + r(t-3) \Rightarrow \int y(t)\delta(t+3)dt = \; y(-3) = 0$

EEE 203, HW 2 **NAME:__SOLUTIONS______**

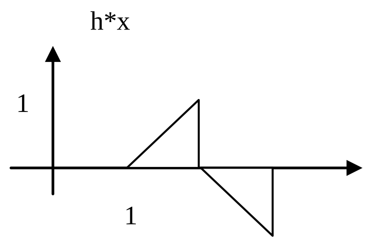
Problem 1. Compute the convolution *h*x* when *x(t)*, *h(t)* are as shown below. Express both signals in terms of elementary functions and use both an analytical and a "graphical" approach



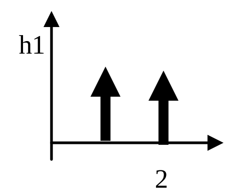


$$x(t) = r(t) - r(t-1) - u(t-1), \quad h(t) = \delta(t-1) - \delta(t-2)$$

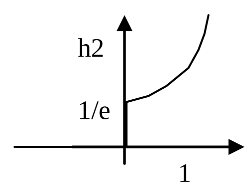
$$(h * x)(t) = x(t-1) - (t-2) = r(t-1) - r(t-2) - u(t-2) - r(t-2) + r(t-3) + u(t-3)$$



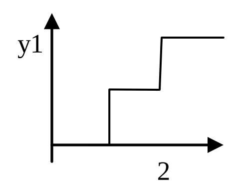
Problem 2. Consider the filters



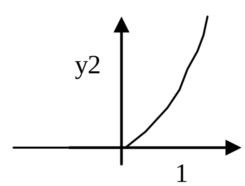
1. $y(t) = x(t-1) + x(t-2)$
2. $y(t) = \int_{-\infty}^{t+1} e^{-\tau+t} x(\tau-1) d\tau$



1. Find and graph their impulse responses.
   1. $h(t) = \delta(t-1) + \delta(t-2)$
   2. $h(t) = \int_{-\infty}^{t+1} e^{-\tau+t} \delta(\tau-1) d\tau = e^{t-1} \int_{-\infty}^{t+1} \delta(\tau-1) d\tau = e^{t-1} u(t)$



2. Find and graph their step responses.
   1. $y(t) = u(t-1) + u(t-2)$
   2. $y(t) = \int_{-\infty}^{t+1} e^{-\tau+t} u(\tau-1) d\tau = \int_{-\infty}^{t} e^{\tau-1} u(\tau) d\tau = \int_{0}^{t} e^{\tau-1} d\tau \, u(t) = \frac{1}{e}[e^{t} - 1]u(t)$

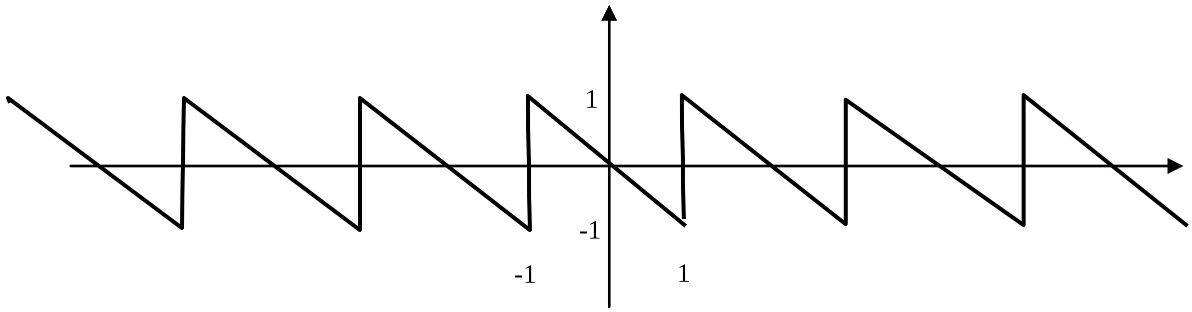


3. Which filters are causal? (Justify)
   1. Is causal, h(t) = 0 for t < 0.
   2. Is causal, h(t) = 0 for t < 0.

4. Which filters are stable? (Justify)
   1. Is stable, $\int |h| = 2 < \infty$
   2. Is not stable, $\int |h|$ diverges. Also, the step response (bounded input) is unbounded.

EEE 203, HW 3 **NAME:__SOLUTIONS_____**

Problem 1: Let *x(t)* be the periodic signal shown in the figure below (sawtooth wave). Compute the coefficients a_k of the Fourier series expansion of *x(t).*



The derivative of x is $\frac{dx}{dt} = -1 + \sum_n 2\delta(t - 2n + 1)$. From the tables, and using the time-shift property, the Fourier series (FS) coefficients of the impulse train are $a_k = \frac{2}{2}(e^{jk\pi}), \forall k$. Then, the FS coefficients of x, say b_k are given by $b_k = \frac{1}{jkw_0} a_k,\ k \neq 0$, $w_0 = \frac{2\pi}{2}$. Thus, $b_k = \frac{e^{jk\pi}}{jk\pi},\ k \neq 0$.

For k = 0, we compute the FS coefficient directly from the definition, $b_0 = \frac{1}{T}\int_T x(t)dt = 0 \Rightarrow b_0 = 0$.

Problem 2: Let *X(jw)* be the Fourier transform of $x(t) = e^{-|t|}$. Find *X(j0)* and $\int_{-\infty}^{\infty} jwX(jw)dw$.

From the definition of the Fourier transform and its inverse, $X(j0) = \int x(t)e^{-j0t}dt = \int e^{-|t|}dt \Rightarrow X(j0) = 2$.

On the other hand, $\frac{dx}{dt}(0) = \frac{1}{2\pi}\int \left(jwX(jw)\right)e^{jw0}dw \Rightarrow \int jwX(jw)dw = 2\pi\frac{dx}{dt}(0) \Rightarrow$

$$\int jwX(jw)dw = 0$$

Problem 3:

Consider the filter with impulse response $h(t) = e^{-(t+1)}u(t-1)$.

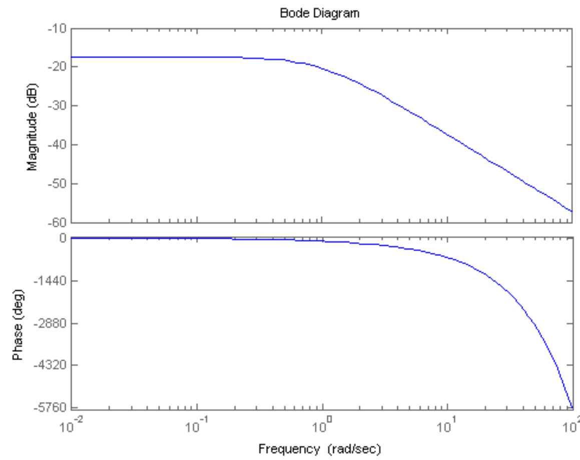
1. Find the transfer function and sketch the Bode Plot
2. Find the Fourier transform of the output when $x(t) = e^{-t}u(t)$ and when $x(t) = \sin(-t)$
3. Find the output when $x(t) = e^{-t}u(t)$ (a) using convolution, and (b) taking the inverse Fourier transform of your answer to part 2.

$$h(t) = e^{-(t+1)}u(t-1) = e^{-2}e^{-(t-1)}u(t-1) \Rightarrow H(jw) = \frac{e^{-2}e^{-jw}}{jw+1}$$

$|H(jw)| = \frac{e^{-2}}{\sqrt{w^2+1}}$, $\angle H(jw) = -w - \tan^{-1}\frac{w}{1}$

In MATLAB,

```
>> H=tf(exp(-2),[1 1])
>> H.iodelay=1
>> bode(H)
```



2.

$$\text{Exponential input}: F\{x\} = \frac{1}{jw+1}, \quad Y(jw) = H(jw)X(jw) = \frac{e^{-2}e^{-jw}}{(jw+1)\ (jw+1)}$$

$$\text{Sinusoid input}: F\{x\} = F\{\sin(-t)\} = \frac{\pi}{j}[\delta(w+1) - \delta(w-1)], Y(jw) = H(jw)X(jw) =$$

$$= -\frac{\pi}{j}e^{-2}\left[\frac{e^{-j}}{j+1}\delta(w-1) - \frac{e^{j}}{-j+1}\delta(w+1)\right] \text{(evaluate H(jw) at the sin frequency)}$$

3. $$y(t) = (h*x)(t) = e^{-2}\int_{-\infty}^{\infty} e^{-(t-1-\tau)}u(t-1-\tau)e^{-\tau}u(\tau)d\tau = e^{-2}e^{-(t-1)}u(t-1)\int_{0}^{t-1} 1d\tau =$$

$$= e^{-2}e^{-(t-1)}(t-1)u(t-1)$$

$$= F^{-1}\{Y(jw)\} = F^{-1}\left\{\frac{e^{-2}e^{-jw}}{(jw+1)\ (jw+2)}\right\} = e^{-2}F^{-1}\left\{\frac{1}{(jw+1)^2}\right\}\Bigg|_{t-1} =$$

$$= e^{-2}[te^{-t}u(t)|_{t-1}] = e^{-2}e^{-(t-1)}(t-1)u(t-1)$$

EEE 203, HW 4 **NAME:__SOLUTIONS____**

Problem 1:
Find the largest sampling interval T_s to allow perfect reconstruction of the signals:
(NOTE: h*x denotes convolution of h and x)

1. $\dfrac{\sin 2t}{4t} * \dfrac{\cos 5t}{3}$
2. $\dfrac{\sin t}{6t} + \dfrac{\sin t}{3}$
3. $\dfrac{\sin 3t}{t} * \sin 2t$

Using the shortcut method, we compute the Nyquist rates (2 x max signal frequency) of the individual signals and then estimate the Nyquist rate of the composite signal:

1. $w_{N1} = 4, w_{N2} = 10 \Rightarrow w_N = \min(w_{N1}, w_{N2}) = 4 \Rightarrow T_s = \frac{2\pi}{w_N} = \frac{\pi}{2}$
2. $w_{N1} = 2, w_{N2} = 2 \Rightarrow w_N = \max(w_{N1}, w_{N2}) = 2 \Rightarrow T_s = \pi$
3. $w_{N1} = 6, w_{N2} = 4 \Rightarrow w_N = \min(w_{N1}, w_{N2}) = 4 \Rightarrow T_s = \frac{2\pi}{4} = \frac{\pi}{2}$

> Note: In fact, a direct computation of #1 shows that the resulting signal is 0, hence any sampling rate can be used. This is consistent with our understanding of the shortcut method producing conservative estimates of the sampling rate.

Problem 2:
Design a sampling system for a measuring a signal with power mostly below 500Hz. Suppose that your sampling frequency cannot be higher than 2kHz and there is noise present between 2-3kHz. Include a suitable anti-aliasing filter (AAF) to reject unwanted noise and determine the parameters of the reconstruction filter. How would you modify the system if you cannot built a good enough analog AAF but you can sample faster?

Since most of the interest is in frequencies below 500Hz, the corresponding Nyquist rate is 1kHz, so our 2kHz sampling frequency is sufficient to provide a solution (the faster, the better). It is also essential to include an AAF to reject the 2-3kHz noise that would cause aliasing. On the other hand, in this problem we would need to balance signal distortion (for which the cutoff frequency should be closer to 1kHz) and noise attenuation (for which the cutoff should be closer to 500Hz). The reconstruction filter should have amplitude T = 0.0005 (2kHz sampling) and cutoff frequency 1kHz. This would provide a more flat DAC transfer function.

Alternatively, if we could implement a faster sampling rate, we could afford a less efficient AAF. In this case, and in order to maintain the same final data rate (2kHz) we would need to include a Discrete time lowpass filter in and downsample the filtered data to the 2kHz rate.

EEE 203, HW 5 **NAME:________________**

Problem 1: Consider the continuous time causal filter with transfer function

$$H(s) = \frac{s}{(s-1)(s+1)}$$

Compute the response of the filter to x(t) = 2u(t-2).

To account for the delay, we will use LTI properties and compute the response to 2u(t) first, and then shift by 2.

$$y_s(t) = L^{-1}\left\{\frac{s}{(s-1)(s+1)} \cdot \frac{2}{s}\right\} = L^{-1}\left\{\frac{1}{(s-1)} + \frac{-1}{s+1}\right\} = e^{t}u(t) - e^{-t}u(t)$$

$$=> y(t) = e^{t-2}u(t-2) - e^{-t+2}u(t-2)$$

Problem 2: Consider the continuous time causal filter described by the differential equation

$$2\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y = 3x - \frac{dx}{dt}$$

Compute the steady-state response of the filter to x(t) = sin(3t)u(t) + 3u(t-3).

$$H(s) = \frac{-s+3}{2s^2+3s+1}$$

The system is stable (transfer function poles or roots of the denominator are $(-1, -0.5)$ are in the LHP, i.e., have negative real parts), hence the steady-state response is well-defined. The steady-state part of the input is sin(3t) + 3, so the steady-state output will also two components

$$y_{ss}(t) = |H(j3)| \sin(3t + \angle[H(j3)]) + H(0)3$$

$$y_{ss}(t) = 0.22 \sin(3t + 163^o) + 9$$

EEE 203, HW 6 **NAME:________________**

Problem 1: Consider the causal filter described by the difference equation
$$y[n+1] = -2y[n] + 4x[n-1]$$
1. Determine the transfer function

2. Compute the response of the filter to $x[n] = u[n]$

Problem 2: Compute the steady-state response of the following discrete-time, causal filters to $x[n] = u[n-12]$:

$$H_1(z) = \frac{z+1}{(z-1)(z+0.5)},$$

$$H_2(z) = \frac{z-1}{(z+0.2)(z+0.8)},$$

$$H_3(z) = \frac{z}{(z-0.5)(z-0.8)}$$

$$H_4(z) = \frac{0.5z+0.5}{(z)(z)}$$