EEE203, TEST 3. **NAME:__SOLUTIONS__________**

30', closed books and notes, calculators and transform tables allowed

Problem 1: Consider the filter with impulse response $h(t) = e^{-t-1}u(t-1)$. Find the Fourier transform of the output Y(jw) when $x(t) = e^{-2t}u(t)$

$$h(t) = e^{-(t-1)}e^{-2}u(t-1) => H(j\omega) = \frac{e^{-2}e^{-j\omega}}{j\omega+1}$$

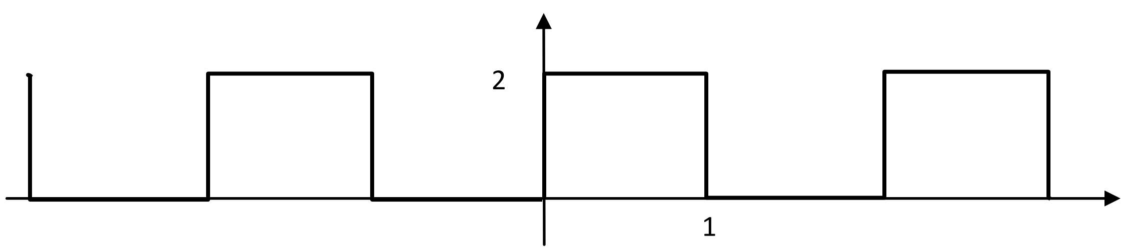
$$X(j\omega) = \frac{1}{j\omega+2}$$

$$=> \ Y(j\omega) = H(j\omega)X(j\omega) = \frac{e^{-2}e^{-j\omega}}{j\omega+1} . \frac{1}{j\omega+2}$$

Problem 2: Consider the continuous time causal filter described by the differential equation $\frac{dy}{dt} + 3y = 2x$. Find the response of the filter to *x*(*t*) = *cos*(*t*).

$$H(j\omega) = \frac{2}{j\omega+3} = \frac{2}{\sqrt{\omega^2+3^2}}\angle - \text{atan}\frac{\omega}{3} => y(t) = |H(j1)|\cos\left(t + \angle H(j1)\right) = \frac{2}{\sqrt{(10)}}\cos\left(t - \text{atan}\left(\frac{1}{3}\right)\right)$$

Problem 3: Let *x(t)* be the periodic signal shown in the figure below (square wave with offset).



Compute the coefficients a_k of the Fourier series expansion of *x(t).*

$$T = 2, \ \omega_0 = \pi, \ T_1 = \frac{1}{2}, \ t_0 = \frac{1}{2}, \ \text{amplitude} = \ 2$$

$$a_k = \frac{2\sin\left(k\omega_0 T_1\right)}{k\pi}e^{-jk\omega_0 t_0} = \frac{2\sin\left(\frac{k\pi}{2}\right)}{k\pi}e^{-\frac{jk\pi}{2}} = \frac{2\sin\left(\frac{k\pi}{2}\right)}{k\pi}\left(\cos\frac{k\pi}{2} - j\sin\frac{k\pi}{2}\right) = \frac{-2j\sin^2\left(\frac{k\pi}{2}\right)}{k\pi}, \ k \neq 0$$

$$a_0 = \frac{1}{T}\int x = \frac{1}{2}(1 \cdot 2) = 1$$