

Problem 1:

Find the largest sampling interval T_s to allow perfect reconstruction of the signals:

(NOTE: $h*x$ denotes convolution of h and x)

$$1. \cos(2\pi t) * \frac{e^{-3t}u(t-1)}{t} \quad w_{NYQ1} = 4\pi = 12.56 \left(\frac{rad}{s}\right), \quad w_{NYQ2} = \infty$$

$$\Rightarrow w_{NYQ} = \max(w_{NYQ1}, w_{NYQ2}) = 12.56 \Rightarrow T_s = 0.5sec$$

$$2. \frac{\sin(t)}{t} \cdot \frac{\sin(2\pi t)}{\pi t} \quad w_{NYQ1} = 2 \times 1 = 2, \quad w_{NYQ2} = 2 \times 2\pi = 12.56 \left(\frac{rad}{s}\right)$$

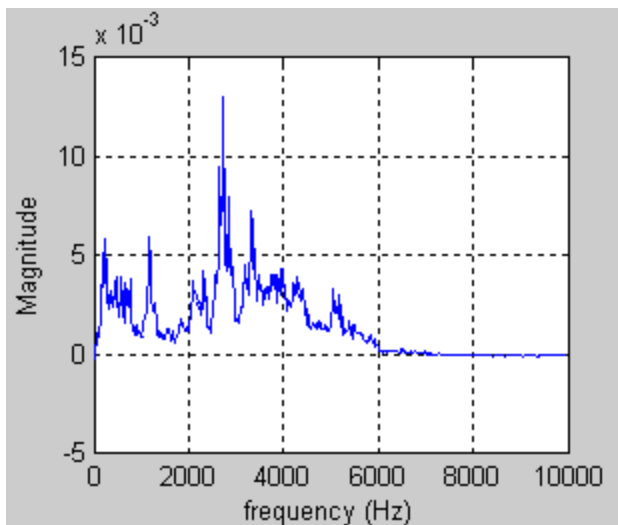
$$\Rightarrow w_{NYQ} = w_{NYQ1} + w_{NYQ2} = 14.56 \Rightarrow T_s = \frac{2\pi}{14.56} = 0.43sec$$

$$3. \cos(2t) - \sin(t) \quad w_{NYQ1} = 2 \times 2 = 4 \left(\frac{rad}{s}\right), \quad w_{NYQ2} = 2 \times 1 = 2 \left(\frac{rad}{s}\right)$$

$$\Rightarrow w_{NYQ} = \max(w_{NYQ1}, w_{NYQ2}) = 4 \Rightarrow T_s = \frac{2\pi}{4} = 1.57sec$$

Problem 2:

The frequency spectrum of a vibration signal is shown in the figure below. We would like to sample and analyze the spectral peaks around 1kHz with a sampling rates of 2kHz. Comment on the feasibility of this objective and describe the ideal components that should be used in such a sampling system.



With 2kHz sampling it is feasible to sample and reconstruct (and therefore, analyze) frequencies of up to 1kHz. Since the signal contains significant power between 2 and 6kHz, we would need an anti-aliasing filter to eliminate them before sampling. The ideal components required are:

1. AAF 1kHz
2. Sampling 2kHz
3. Analysis tools

Note that for practical implementation, the AAF should be high order with sharp roll-off in order to attenuate virtually all frequencies past 1kHz to avoid aliasing but preserve frequencies

below 1kHz without much distortion.