EEE 203, Test 5 **NAME:__SOLUTIONS___**

CLOSED BOOK & NOTES. TRANSFORM TABLES ALLOWED. 30'

Problem 1: Consider the continuous time causal filter with transfer function

$$H(s) = \frac{1}{(s+1)(s+2)}$$

Compute the response of the filter to x(t) = u(t) – u(t-2).

We compute the step response first:

$$y_s(t) = L^{-1}\left\{\frac{1}{(s+1)(s+2)} \cdot \frac{1}{s}\right\} = L^{-1}\left\{\frac{-1}{(s+1)} + \frac{0.5}{s+2} + \frac{0.5}{s}\right\}$$
$$= -e^{-t}u(t) + 0.5e^{-2t}u(t) + 0.5u(t)$$

Then we construct the overall response using linearity and Time-Invariance

$$=> y(t) = y_s(t) - y_s(t-2)$$
$$= -e^{-t}u(t) + 0.5e^{-2t}u(t) + 0.5u(t)$$
$$+e^{-t+2}u(t-2) - 0.5e^{-2t+4}u(t-2) - 0.5u(t-2)$$

Problem 2: Consider the continuous time causal filter described by the differential equation

$$2\frac{d^2y}{dt^2} + 1\frac{dy}{dt} + 1y = \frac{dx}{dt} - 2x$$

Compute the steady-state response of the filter to x(t) = sin(t)u(t) + cos(2t)u(t-2).

$$H(s) = \frac{s-2}{2s^2+s+1}$$

The system is stable (transfer function poles or roots of the denominator are $(-0.25 \pm 0.661j)$ and they are in the LHP, i.e., have negative real parts), hence the steady-state response is well-defined. The steady-state part of the input is sin(t) + cos(2t), so the steady-state output will also two components

$$y_{ss}(t) = |H(j1)|\sin(t + \angle[H(j1)]) + |H(j2)|\cos(2t + \angle[H(j2)])$$

$$y_{ss}(t) = 1.58\sin(t + 18.4^o) + 0.389\cos(2t - 29^o)$$