EEE 203, Test 6 **NAME:____SOLUTIONS_____**

Problem 1: Consider the causal filter described by the difference equation

$$y[n+2] = -0.3y[n] + 0.3x[n-1] - x[n-2]$$

1. Determine the transfer function

$$H_1(z) = \frac{0.3z - 1}{(z^2)(z^2 + 0.3)}$$

2. Compute the response of the filter to $x[n] = u[n]$

Starting with zero initial conditions, its step response is

$$Y(z) = Z^{-1}\left\{\frac{0.3z-1}{(z^2)(z^2+0.3)} \bullet \frac{z}{z-1}\right\} = Z^{-1}\left\{\frac{0.33}{z} + \frac{-0.54}{z-1} + \frac{-1.4-0.49j}{z-0.55j} + \frac{-1.4+0.49j}{z+0.55j}\right\}$$

$$y(n) = 0.33\delta(n-1) - 0.54u(n-1) - 2\Re[(1.4+0.49j)(0.55j)^{n-1}]u(n-1)$$

$$y(n) = 0.33\delta(n-1) - 0.54u(n-1) - \Re[2.96e^{0.34j}(0.55)^{n-1}e^{1.57(n-1)j}]u(n-1)$$

$$y(n) = 0.33\delta(n-1) - 0.54u(n-1) - 2.96(0.55)^{n-1}\cos(1.57(n-1)+0.34)u(n-1)$$

Problem 2: Compute the steady-state response of the following discrete-time, causal filters to $x[n] = 3u[n-12]$:

$H_1(z) = \dfrac{z-1}{(z)(z-0.5)}$, The steady state input is 3, the transfer function is stable because its poles have magnitude 0, 0.5 (less than one), so the steady state output is 3H_1(1) = 0.

$H_2(z) = \dfrac{z}{(z-1)(z-0.5)}$, The transfer function is not stable so the steady state response is not well-defined.

Problem 3: Compute the steady-state response of the following discrete-time, causal filters to $x[n] = \cos(0.1n)u[n-15]$:

$H_3(z) = \dfrac{z}{(z+0.1)(z-0.5)}$ The system is stable because its poles have magnitude less than one. The steady-state response is given by the formula

$$y[n] = |H_3(e^{j0.1})| \cos\left(0.1n + \angle H_3(e^{j0.1})\right) =$$

$$= \left|\frac{\cos 0.1 + j\sin 0.1}{(\cos 0.1 + 0.1 + j\sin 0.1)(\cos 0.1 - 0.5 + j\sin 0.1)}\right| \cos\left(0.1n + \angle\frac{\cos 0.1 + j\sin 0.1}{(\cos 0.1 + 0.1 + j\sin 0.1)(\cos 0.1 - 0.5 + j\sin 0.1)}\right)$$

$$= \frac{1}{|1.095 + j0.1||0.495 + j0.1|}\cos\left(0.1n + 0.1 - \operatorname{atan}\frac{0.1}{1.095} - \operatorname{atan}\frac{0.1}{0.495}\right)$$

$$= 1.8\cos(0.1n - 0.19) = 1.8\cos(0.1n - 10.9^o)$$