EEE 303 HW # 1 SOLUTIONS

Problem 1.5

Since $x(t) = 0$ for $t < 3$, we have that $x(t) = U(t-3)x(t)$.

a. $x(1-t) = x(1-t)U(1-t-3) = x(1-t)U(-t-2)$ which is zero for $t > -2$.

b. $x(1-t) + x(2-t) = x(1-t)U(-t-2) + x(2-t)U(-t-1)$ which is zero for $t > -1$.

c. $x(1-t)x(2-t) = x(1-t)U(-t-2)x(2-t)U(-t-1) = x(1-t)x(2-t)U(-t-2)$ which is zero for $t > -2$.

d. $x(3t) = x(3t)U(3t-3)$ which is zero for $t < 1$.

e. $x(t/3) = x(t/3)U(t/3-3)$ which is zero for $t < 9$.

EEE 303 HW#2 Solutions

1.43

a. Let $\boldsymbol{H}$ denote a TI system such that $y = \boldsymbol{H}[x]$ and suppose that x is periodic. That is, $x(t+T)=x(t)$, for all t. Alternatively, in terms of the shift operator $\boldsymbol{T}$, $\boldsymbol{T}_{-T}[x] = x$. Now, for y to be periodic, we should also have $\boldsymbol{T}_{-T}[y] = y$. Substituting $y = \boldsymbol{H}[x]$, we get

$$
\begin{aligned}
\boldsymbol{T}_{-T}[y] &= \boldsymbol{T}_{-T}\boldsymbol{H}[x] \\
&= \boldsymbol{H}\boldsymbol{T}_{-T}[x] && \text{(TI system, commutes with shifts)} \\
&= \boldsymbol{H}[x] && \text{(x is periodic)} \\
&= y
\end{aligned}
$$

(The derivations hold in discrete time as well.)

b. The zero system ($y = 0$) is an easy but trivial example. A more interesting case is a low-pass filter eliminating the high frequency component of, say, $x(t) = \sin t + \sin 10\pi t$. (More details later.)

EEE 303 HW#3 Solutions

2.20

a. $\int_{-\infty}^{\infty} u_0(t)\cos(t)dt = \int_{-\infty}^{\infty}\delta(t)\cos(t)dt = \int_{-\infty}^{\infty}\delta(t)\cos(0)dt = 1$

b. $\int_0^5 \sin(2\pi t)\delta(t+3)dt = 0$, because $\delta(t+3) = 0$ for t in [0,5].

c. $\int_{-5}^{5} u_1(1-\tau)\cos(2\pi\tau)d\tau = \frac{d[\cos(2\pi t)]}{dt}\Big|_{t=1} = -2\pi\sin(2\pi) = 0$, since 1 is in [-5,5].

Alternatively, using integration by parts and $u_1(t) = \frac{d\delta(t)}{dt}$, we have that $u_1(1-\tau) = -\frac{d}{d\tau}\delta(1-\tau)$. Then,

$$\int_{-5}^{5} u_1(1-\tau)\cos(2\pi\tau)d\tau = -\int_{-5}^{5}\frac{d\delta(1-\tau)}{d\tau}\cos(2\pi\tau)d\tau =$$

$$-\left[\delta(1-t)\cos(2\pi t)\Big|_{-5}^{5}\right] + \int_{-5}^{5}\delta(1-\tau)\frac{d\cos 2\pi\tau}{d\tau}d\tau = -\sin(2\pi 1) = 0$$

where we used the fact that $\delta(1\text{-}5) = \delta(1\text{+}5) = 0$.

2.47

Let $y_0 = h_0 * x_0$. Then,

a. $[h_0(t)]*[2x_0(t)] = 2(h_0 * x_0) = 2y_0$, (linearity of convolution)

b. $[h_0(t)]*[x_0(t) - x_0(t-2)] = (h_0 * x_0)(t) - [h_0(t)]*[x_0(t-2)] = y_0(t) - y_0(t-2)$,

where the first equality follows from linearity and the second from time-invariance.

c. Using time-invariance, $[h_0(t+1)]*[x_0(t-2)] = ([h_0(t)]*[x_0(t-2)])(t+1)$.

But, $[h_0(t)]*[x_0(t-2)] = ([h_0(t)]*[x_0(t)])(t-2) = y_0(t-2)$.

Hence, $[h_0(t+1)]*[x_0(t-2)] = y_0(t+1-2) = y_0(t-1)$.

Alternatively, using the definition of the shift operator T and its commutativity with convolution,

$[h_0(t+1)]*[x_0(t-2)] = (T_{-1}h_0)*(T_2 x_0) = T_{-1}T_2(h_0 * x_0) = T_1 y_0$.

d. There is not enough information to determine the output in this case. To prove this statement we need to find two pairs (h_0,x_0) that produce the same response y_0 and such that their corresponding transformations produce different responses. For this, consider $h_0(t) = U(t)$ (a unit step) and $x_0(t)$ = {a pulse in [0,2] with amplitude ½}. As a second pair, consider $h_0(t) = y_0(t)$, $x_0(t) = \delta(t)$. For the first pair, it follows that $[h_0(t)]*[x_0(-t)] = y_0(t+2)$. But for the second pair, $[h_0(t)]*[x_0(-t)] = y_0(t)$.

e. Reflections are not time-invariant operations so we need to work with the convolution integral itself. Using a transformation of the integration variable $\sigma = -\tau$, we have that $y_0(-t)$ can be expressed as:

$$y_0(-t) = \int_{-\infty}^{\infty} h_0(-t-\tau)x_0(\tau)d\tau = \int_{-\infty}^{\infty} h_0(-t+\sigma)x_0(-\sigma)d\sigma = \int_{-\infty}^{\infty} h(t-\sigma)x(\sigma)d\sigma = (h*x)(t)$$

Notice that **this is not** a simple transformation of the independent variable. That is, we may **not** simply state that since $y_0(t) = h_0(t)*x_0(t)$, then $y_0(-t) = h_0(-t)*x_0(-t)$. Even though the final relationship is true, this statement is wrong as an implication! The error is caused by the bad notation ($h(t)*x(t)$ is nonsense). This error becomes apparent when we consider scaling of the independent variable. That is, using the previous derivation it follows that $y_0(at) = |a|\ [h_0(at)]*[x_0(at)]$ (and **not** $h_0(at)*x_0(at)$, as one might expect!)

f. Using the property of the unit doublet u_1, that $x' = u_1 * x$, we have that

$$y = h*x = h'_0 * x'_0 = (u_1 * h_0)*(u_1 * x_0) = u_1 * u_1 * (h_0 * x_0) = y_0''$$

From the graph of y_0, it follows that $y(t) = ½\ \delta(t) - ½\ \delta(t-2)$.

EEE 303 HW # 4 SOLUTIONS

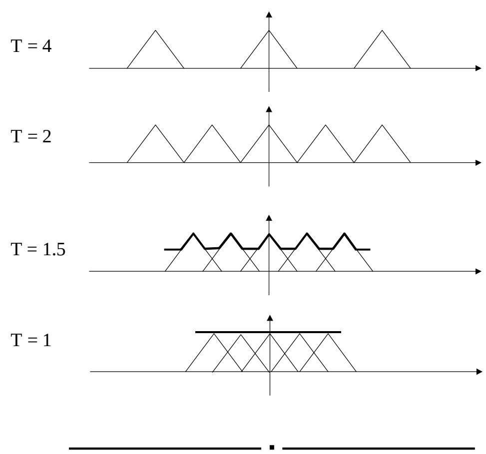
Problem 2.23

Using the linearity and time invariance of convolution, we have that ($\mathcal{T}$ denotes the shift)

$$\begin{aligned} h * x &= h * \left(\sum_k \mathcal{T}_{kT}\delta \right) \\ &= \sum_k h * (\mathcal{T}_{kT}\delta) \\ &= \sum_k \mathcal{T}_{kT}(h * \delta) \\ &= \sum_k \mathcal{T}_{kT}h \end{aligned}$$

So, finally,

$$y(t) = \sum_k h(t - kT)$$



EEE 303 HW#4 Solutions

3.15 The Fourier series coefficients of the output $y(t)$, say b_k are related to the Fourier series coefficients of the input (a_k) by

$$b_k = H(jkw_0)\, a_k$$

where $w_0 = 2\pi / T = 12$. Since $H(jw) = 0$ for $|w| > 100$, it follows that $b_k = 0$ for $|12\,k| > 100$, or $|k| > 100/12$, or $|k| \geq 9$. Since it is given that $x(t) = y(t)$, then $a_k = b_k$ (uniqueness of the FS expansion), implying that a_k must be zero for $|k| \geq 9$.

EEE 303 HW Solutions

3.22.a.a

Consider the derivative of the given function:

$$\frac{dx}{dt}(t) = 1 - 2\sum_{n=-\infty}^{\infty}\delta(t-2n-1)$$

The train of deltas is periodic with period 2. Its Fourier Series expansion can be found either directly (integrals of delta-times-a-function) or by using the tables and applying the shift property. We have,

$$a_k = FS\left\{\sum_n \delta(t-2n-1)\right\} = \tfrac{1}{2}\int_0^2 \delta(t-1)e^{-jkw_0 t}dt = \frac{e^{-jkw_0}}{2}$$

where $w_0 = 2\pi/2 = \pi$. The FS coefficients of the function 1 are 1 if k = 0 and 0 otherwise. Thus, the coefficients of the derivative are

$$b_k = FS\left\{\tfrac{dx}{dt}(t)\right\} = \begin{cases} 1 - e^{-jk\pi} & if\ k = 0 \\ -e^{-jk\pi} & otherwise \end{cases}$$

Next, the FS coefficients of *x(t)*, say c_k, are related to b_k by $b_k = jkw_0 c_k$. This can be solved for c_k except for *k* = 0. This can be found by applying the FS definition directly to *x(t)*. Thus, we find

$$c_k = FS\{x(t)\} = \begin{cases} 0 & if\ k = 0 \\ -e^{-jk\pi}/(jk\pi) & otherwise \end{cases}$$

Notice that the direct computation of c_0 here is essential. Both *x(t)* and *x(t)+const.* have the same derivative and their FS cannot be distinguished after differentiation.

REMARK: Applying Parseval's theorem on *x(t)* we find:

$$\sum_{-\infty}^{\infty}|c_k|^2 = \sum_{k\neq 0}\frac{1}{k^2\pi^2} = 2\sum_{k=1}^{\infty}\frac{1}{k^2\pi^2} = \frac{1}{2}\int_{-1}^{1}t^2 dt = \frac{1}{3}$$

This yields the well-known expression

$$\sum_{k=1}^{\infty}\frac{1}{k^2} = \frac{\pi^2}{6}$$

3.22.a.b

Using the same technique as in a.a, let x_0 be the square wave

$$x_0(t) = \begin{cases} 1 & |t| < 0.5 \\ 0 & otherwise \end{cases} \quad , \quad x_0(t+6) = x_0(t)$$

whose Fourier Series coefficients are given in Table 4.2. Differentiating the given signal, we find

$$\frac{dx}{dt}(t) = x_0(t+1.5) - x_0(t-1.5)$$

Hence, applying the linearity and time-shift properties of the Fourier Series, we get

$$FS\left\{\tfrac{dx}{dt}(t)\right\} = \frac{\sin(k\pi/6)}{k\pi}\left(e^{jk\pi/2} - e^{-jk\pi/2}\right) = 2j\frac{\sin(k\pi/6)}{k\pi}\sin(k\pi/2)$$

Next, using the differentiation property, we find

$$a_k = FS\{x(t)\} = FS\left\{\tfrac{dx}{dt}(t)\right\}/(jkw_0) = 6\frac{\sin(k\pi/6)}{(k\pi)^2}\sin(k\pi/2), \quad k \neq 0$$

while for k = 0, the FS coefficient is found by direct integration as $a_0 = (2+1/2+1/2)/6 = ½$.

EEE 303 HW # 9 SOLUTIONS

Problem 3.16b

$x_2[n]$ is composed of two periodic functions, so we can apply superposition:

$$y_s[n] = H(e^{j0})1 + |H(e^{j3\pi/8})| \sin(n3\pi/8 + \pi/4 + \arg\{H(e^{j3\pi/8})\})$$

But $H(e^{j0}) = 0$ and $H(e^{j3\pi/8}) = 1$, so

$$y_s[n] = \sin(n3\pi/8 + \pi/4)$$

Alternatively, one could write the Fourier series expansion for x_2 and use the filtering property.

Problem 3.32

The set of equations for a_k is

$$\{\phi^{nk}\}_{n,k}\{a_k\}_k = \{x[n]\}_n$$

where $\{.\}_k$ denotes the vector or matrix indexed by k and $\phi = e^{j2\pi/4}$. Expanding the matrices, we get

$$\underbrace{\begin{bmatrix} \phi^0 & \phi^0 & \phi^0 & \phi^0 \\ \phi^0 & \phi^1 & \phi^2 & \phi^3 \\ \phi^0 & \phi^2 & \phi^4 & \phi^6 \\ \phi^0 & \phi^3 & \phi^6 & \phi^9 \end{bmatrix}}_{\Phi} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}$$

The matrix Φ has a very special structure. Its columns are of the form $[1,\ \rho_i,\ \rho_i^2,\ \rho_i^3]^\top$. Such matrices are called Vandermonde matrices and they are nonsingular if the ρ_i's are distinct. In our case $\rho_1 = \phi^0 = 1$, $\rho_2 = \phi^1 = j$, $\rho_3 = \phi^2 = -1$, $\rho_4 = \phi^3 = -j$, which are indeed distinct.

In general, Vandermonde matrices are numerically ill-conditioned except when the ρ_i's are the n-th roots of unity, which is our case. When this happens, the columns of the matrix are orthogonal to each other and the matrix is easy to invert. (Observe that $\sum_k (\phi^{kn_1})^* \phi^{kn_2} = \sum_k \phi^{k(n_2-n_1)}$ which is 0 if $n_2 \neq n_1$. This property is precisely the reason why we can write a simple expression for the Fourier series coefficients a_k.)

Regardless of any simplifications, we can use Matlab (or any other numerical package) to solve the above system of equations. Substituting the numerical values we have

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}$$

and Matlab produces the solution $a_0 = 1/2$, $a_1 = (-1-j)/4$, $a_2 = 1$, $a_3 = (-1+j)/4$.

Needless to say, we arrive at the same result with the Fourier Series equation.

EEE 303 HW # 5 SOLUTIONS

Problem 4.34

a. From the definition of the transfer function as the Fourier transform of the impulse response, we have

$$\frac{Y(jw)}{X(jw)} = \frac{(jw)+4}{(jw)^2+5(jw)+6} \Rightarrow$$
$$[(jw)^2+5(jw)+6]Y(jw) = [(jw)+4]X(jw)$$

Taking the inverse Fourier transform of both sides, we recover the differential equation that describes the input-output relationship for the system

$$\frac{d^2}{dt}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = \frac{d}{dt}x(t) + 4x(t)$$

b. We have that $h(t) = \mathcal{F}^{-1}\{H(jw)\}$; using a partial fraction expansion, the roots of the denominator are $-2, -3$ and

$$H(jw) = \frac{A}{jw+2} + \frac{B}{jw+3}$$

where $A = [(jw+2)H(jw)]|_{jw=-2} = 2$ and $B = [(jw+3)H(jw)]|_{jw=-3} = -1$. From the tables,

$$\mathcal{F}^{-1}\{H(jw)\} = \mathcal{F}^{-1}\left\{\frac{2}{jw+2}\right\} + \mathcal{F}^{-1}\left\{\frac{-1}{jw+3}\right\} = 2e^{-2t}U(t) - e^{-3t}U(t)$$

c. The response of the system to the input $x(t) = e^{-4t}U(t) - te^{-4t}U(t)$ can be found by either a direct evaluation of the convolution of the impulse response with the input or by Fourier transform methods. Using the latter approach, we find the Fourier transform of $x(t)$ as

$$X(jw) = \frac{1}{jw+4} - \frac{1}{(jw+4)^2}$$

Then, $Y(jw) = H(jw)X(jw)$ so,

$$\begin{aligned}
Y(jw) &= \frac{jw+4}{(jw+2)(jw+3)}\left(\frac{1}{jw+4} - \frac{1}{(jw+4)^2}\right) \\
&= \frac{1}{(jw+2)(jw+3)} - \frac{1}{(jw+2)(jw+3)(jw+4)} \\
&= \frac{1}{(jw+2)} + \frac{-1}{(jw+3)} - \frac{1/2}{(jw+2)} - \frac{-1}{(jw+3)} - \frac{1/2}{(jw+4)} \\
&= \frac{1/2}{(jw+2)} - \frac{1/2}{(jw+4)}
\end{aligned}$$

It is now straightforward to evaluate the time response:

$$y(t) = \mathcal{F}^{-1}\{Y(jw)\} = \frac{1}{2}\left[e^{-2t}U(t) - e^{-4t}U(t)\right]$$

Notice the cancellation of the pole at -3 by the zero of $X(jw) = (jw+3)/(jw+4)$. Also, the transfer function zero at -4 cancelled one of the poles of $X(jw)$ and we did not need to evaluate the partial fraction expansion for multiple poles.

Problem 4.35

a. For the transfer function

$$H(jw) = \frac{a-jw}{a+jw}$$

we have that $|H(jw)| = \sqrt{a^2+(-w)^2}/\sqrt{a^2+w^2} = 1$. Such transfer functions are called "all-pass." Further, $\angle H(jw) = \tan^{-1}(-w/a) - \tan^{-1}(w/a) = -2\tan^{-1}(w/a)$.

b. The impulse response of the system is the inverse Fourier transform of its transfer function:

$$h(t) = \mathcal{F}^{-1}\left\{H(jw)\right\} = \mathcal{F}^{-1}\left\{\frac{a + a - a - jw}{a + jw}\right\} = \mathcal{F}^{-1}\left\{\frac{2a}{a + jw} - 1\right\} = 2ae^{-at}U(t) - \delta(t)$$

c. The response of the system to the input

$$x(t) = \cos(t/\sqrt{3}) + \cos(t) + \cos(\sqrt{3}t)$$

can be evaluated through Fourier transforms. $X(jw)$ will contain 6 impulses and so will the product $X(jw)H(jw)$. Then the inverse transform will contain 6 complex exponentials, but the simplification of that formula is quite tedious.

Alternatively, using the sinusoid response formula and for $a = 1$,[1]

$$\begin{aligned} y(t) &= \cos(t/\sqrt{3} - 2\tan^{-1}(1/\sqrt{3})) + \cos(t - 2\tan^{-1}(1)) + \cos(\sqrt{3}t - 2\tan^{-1}(\sqrt{3})) \\ &= \cos(t/\sqrt{3} - \pi/3) + \cos(t - \pi/2) + \cos(\sqrt{3}t - 2\pi/3) \end{aligned}$$

The following figure shows the response of the all-pass filter to the three cosines. For comparison, the response of the filter to the input is $x(t)U(t)$ is also included. In the latter case, notice that the appearance of the $\delta(t)$ in the impulse response causes the initial output to be $y(0+) = -x(0+) = -3$.

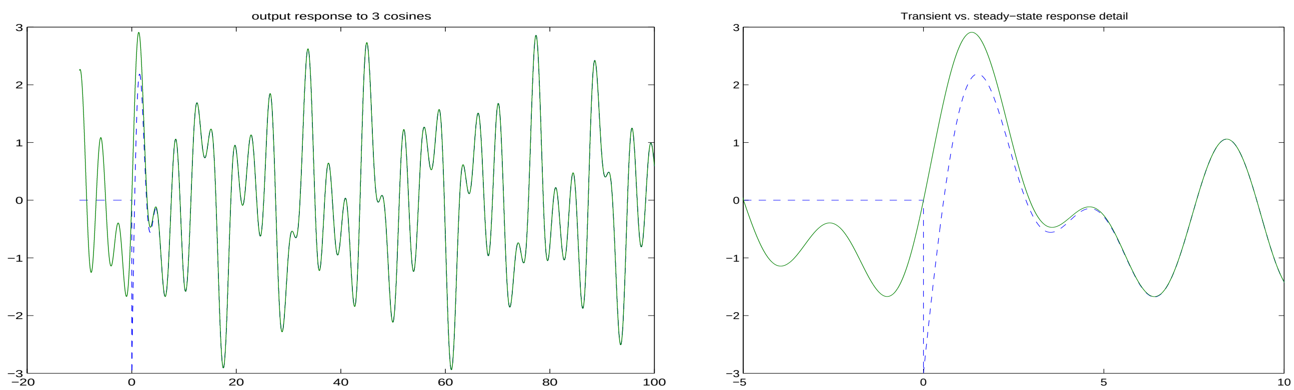


Figure 1: Left: Output response to the three cosines. Right: Detail showing the difference between the steady-state response and the transient response. (Steady-state response: solid line.)

[1]The response of $H(jw)$ to $\cos(w_0t)$ is $|H(jw_0)|\cos(w_0t + \angle H(jw_0))$.

EEE 303 HW # 9 SOLUTIONS

Problem 5.10

Let $A = \sum_0^\infty n\frac{1}{2^n}$ and define the sequence $x[n] = \frac{1}{2^n}u[n]$. Then $X(e^{jw}) = \frac{1}{1-e^{-jw}/2}$. The transform of the sequence $nx[n]$ is $j\frac{dX(e^{jw})}{dw} = \frac{e^{-jw}/2}{(1-e^{-jw}/2)^2}$. Then, $A = \mathcal{F}\{nx[n]\}|_{e^{j0}} = \frac{1/2}{(1-1/2)^2} = 2$

Problem 5.19

By inspection, the transfer function is

$$H(z) = \frac{1}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}$$

where $z = e^{jw}$.

The poles (roots of denominator) are $z = 1/2$ and $-1/3$. Taking the Partial Fraction Expansion we have

$$H(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{3}z^{-1})} = \frac{\frac{3}{5}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{2}{5}}{1 + \frac{1}{3}z^{-1}}$$

Using the tables to find the inverse Fourier, we get

$$h[n] = \frac{3}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(-\frac{1}{3}\right)^n u[n]$$

Note: The impulse response can also be computed directly from the recursion. First, since the system is linear, its response to the zero input sequence is zero. Furthermore since it is causal, then its response to $\delta[n]$ must be identical to the zero-input response up to the index $n = 0$. So, $y[n] = 0$, for $n < 0$. Then,

$$\begin{aligned} y[0] &= 0 + 0 + x[0] = 1 \\ y[1] &= 1/6 - 0 + 0 = 1/6 \\ y[2] &= (1/6)(1/6) + 1/6 + 0 = 7/36 \\ &\vdots \end{aligned}$$

which is the same as the previous analytical computations.

EEE 303 HW # 6 SOLUTIONS

Problem 6.9

The filter is stable so the steady-state response is well-defined. The steady-state part of the step is the constant function 1. The filter transfer function is $H(jw) = \frac{2}{jw+5}$ which at $w = 0$ is 2/5. Therefore, the steady-state part of the step response is the constant function $(2/5)(1) = 2/5$, which is also the final value of $y_s(t),\ t \to \infty$.

The step response is $Y_s(s) = \frac{2}{s+5}\frac{1}{s}$. Taking the inverse Laplace via partial fraction expansion we find:

$$y_s(t) = \mathcal{L}^{-1}\left\{\frac{2}{s+5}\frac{1}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{2/5}{s} + \frac{-2/5}{s+5}\right\} = \frac{2}{5}u(t) - \frac{2}{5}e^{-5t}u(t) = \frac{2}{5}(1 - e^{-5t})u(t)$$

The same result is obtained by using the Fourier transform, but notice the difference in the transform of the step function:

$$\begin{aligned}
Y_s(jw) &= \frac{2}{jw+5}\left(\frac{1}{jw} + \pi\delta(w)\right) = \frac{2}{5}\pi\delta(w) + \frac{2}{jw+5}\frac{1}{jw} \\
y_s(t) &= \mathcal{F}^{-1}\{Y_s(jw)\} = \mathcal{F}^{-1}\left\{\frac{2}{5}\pi\delta(w) + \frac{2}{jw+5}\frac{1}{jw}\right\} = \mathcal{F}^{-1}\left\{\frac{2}{5}\pi\delta(w) + \frac{2/5}{jw} + \frac{-2/5}{jw+5}\right\} \\
&= \mathcal{F}^{-1}\left\{\frac{2}{5}\left(\frac{1}{jw} + \pi\delta(w)\right) + \frac{-2/5}{jw+5}\right\} = \frac{2}{5}u(t) - \frac{2}{5}e^{-5t}u(t)
\end{aligned}$$

As $t \to \infty$, this expression yields $y_s(\infty) = 2/5$, as found in the first part. Then, the time t_0 at which $y_s(t_0) = y_s(\infty)(1 - e^{-2})$ is $t_0 = 2/5$.

Problem 6.12

The transfer function of the cascade connection is $H(jw) = H_1(jw)H_2(jw)$. Hence, $H_2(jw) = \frac{H(jw)}{H_1(jw)}$. Taking the magnitude and log of both sides,

$$\log_{10}|H_2(jw)| = \log_{10}|H(jw)| - \log_{10}|H_1(jw)| \Rightarrow |H_2(jw)|_{dB} = |H(jw)|_{dB} - |H_1(jw)|_{dB}$$

Thus the straight-line approximation of $|H_2(jw)|_{dB}$ is described as follows:

for $w < 1$**:** constant with magnitude (-20)-(6) = -26 dB;

for $< 1w < 8$**:** straight line with slope (0)-(20) = -20 dB/dec;

for $8 < w < 40$**:** straight line with slope (-40)-(0) = -40 dB/dec;

for $40 < w$**:** straight line with slope (-40)-(-20) = -20 dB/dec;

EEE 303 HW # 6 SOLUTIONS

Problem 6.27

(a.) Using the differentiation property of the Fourier transform (or Laplace) the transfer function of the system is:

$$H(jw) = \frac{1}{jw+2}$$

or, $H(s) = 1/(s+2)$. The Laplace version is more advantageous since in the subsequent derivations it is convenient to keep jw as one argument.

With $s = jw$, the frequency response of this transfer function has the following magnitude and phase:

$$\begin{aligned} |H(jw)| &= \frac{1}{\sqrt{w^2+2^2}} \\ \angle H(jw) &= \tan^{-1}(w/2) \end{aligned}$$

These are easy to plot; in MATLAB, the relevant command is `bode([1],[1 2]);` (see also `help bode`).

(b.) For the group delay, differentiate $-\tan^{-1}(w/2)$ with respect to w.

(c. & d.) When $x(t) = e^{-t}\mathcal{U}(t)$, $\mathcal{F}\{x\} = 1/(s+1)$, with $s = jw$. Using the convolution property,

$$Y(jw) = \left.\frac{1}{(s+2)(s+1)}\right|_{s=jw}$$

For its partial fraction expansion (PFE), we need to find A, B such that

$$Y(s) = \frac{A}{(s+2)} + \frac{B}{(s+1)}$$

The values of A, B can be computed using the formulas in the Appendix, or simply by converting the partial fractions into one fraction and equating the coefficients of equal powers of s in the numerator. (This is effective when only a few terms are involved.)

$$A = \left.\frac{1}{(s+1)}\right|_{s=-2} \quad ; \quad B = \left.\frac{1}{(s+2)}\right|_{s=-1}$$

Now, the time response can be computed easily by using the tables:

$$y(t) = (1)e^{-t}\mathcal{U}(t) + (-1)e^{-2t}\mathcal{U}(t)$$

(e-i.) Here $X(jw) = \frac{1+jw}{2+jw}$, so

$$Y(s) = \frac{s+1}{(s+2)^2} = \frac{A_1}{(s+2)} + \frac{A_2}{(s+2)^2}$$

Converting the PFE back to one fraction we have that

$$A_1 s + 2A_1 + A_2 = s + 1 \Rightarrow A_1 = 1,\ A_2 = -1$$

Hence, $y(t) = (1)e^{-2t}\mathcal{U}(t) + (-1)te^{-2t}\mathcal{U}(t)$.

Alternatively, since the fraction involves only one pole, we have that $y(t) = y_0(t) + \frac{d}{dt}y_0(t)$, where $y_0(t) = \mathcal{F}^{-1}\left\{1/(jw+2)^2\right\} = te^{-2t}\mathcal{U}(t)$. (Verify this!)

(e-ii.) Here $Y(s) = \frac{s+2}{(s+2)(s+1)} = \frac{1}{(s+1)}$ and, clearly, $y(t) = e^{-t}\mathcal{U}(t)$.

(e-iii.) Here

$$Y(s) = \frac{1}{(s+2)^2(s+1)} = \frac{A}{(s+1)} + \frac{B_1}{(s+2)} + \frac{B_2}{(s+2)^2}$$

Using the PFE formulas, we find $A = 1$, $B_1 = -1$, $B_2 = -1$.
Hence, $y(t) = (1)e^{-t}\mathcal{U}(t) + (-1)e^{-2t}\mathcal{U}(t) + (-1)te^{-2t}\mathcal{U}(t)$.

EEE 303 HW # 6 SOLUTIONS

Problem 7.4

The Nyquist rate of $x(t)$ is w_0, so $x(t)$ is bandlimited in $(-w_0/2,\ w_0/2)$. In other words, the maximum frequency where $x(t)$ has energy is $w_M = w_0/2$.

1. $\mathcal{F}\{x(t-1)\} = e^{-jw}\mathcal{F}\{x(t)\}$ is also bandlimited in $(-w_M, w_M)$. Hence, $\mathcal{F}\{x(t-1)+x(t)\}$ is bandlimited in $(-w_M, w_M)$, and the Nyquist rate of $x(t-1)+x(t)$ is $2w_m = w_0$.

2. $\mathcal{F}\left\{\frac{dx}{dt}\right\} = jw\mathcal{F}\{x(t)\}$ which is again bandlimited in $(-w_M, w_M)$ and, hence, the Nyquist rate of dx/dt is w_0.

Notice that this result hinges on the assumption that $X(jw)$ is zero for $|w| > w_M$. This can be deceptive in a practical situation where the Nyquist rate is defined in terms of the signal "bandwidth," i.e., the frequency beyond which the energy content of the signal is small. In such a case, differentiation of the signal will increase the magnitude (and energy) of the signal at high frequencies and will change the effective Nyquist rate.

3. Let $y(t) = x^2(t)$. Then $\mathcal{F}\{y\} = \frac{1}{2\pi}\mathcal{F}\{x\} * \mathcal{F}\{x\}$. Since $\mathcal{F}\{x\}$ is bandlimited in $(-w_M, w_M)$, we have that $\mathcal{F}\{y\}$ is bandlimited in $(-2w_M, 2w_M)$; hence the Nyquist rate of $y(t)$ is $2w_0$.

4. Let $y(t) = x(t)\cos(w_0 t)$. Then $\mathcal{F}\{y\} = \frac{1}{2\pi}\mathcal{F}\{x\}\mathcal{F}\{\cos w_0 t\} = \frac{1}{2}[X(j(w-w_0)) + X(j(w+w_0))]$. Hence, $\mathcal{F}\{y\}$ is bandlimited in $(-w_0 - w_M, w_0 + w_M) = (-3w_M, -3w_M)$ and the Nyquist rate of $y(t)$ is $6w_M = 3w_0$.

Problem 7.22

Let $x_1(t)$ be bandlimited in $(-1000\pi, 1000\pi)$ and $x_2(t)$ be bandlimited in $(-2000\pi, 2000\pi)$.

a. Let $y(t) = (x_1 * x_2)(t)$. Then $\mathcal{F}\{y\} = \mathcal{F}\{x_1\}\mathcal{F}\{x_2\}$ which is bandlimited in the intersection of the two frequency intervals, that is $(-1000\pi, 1000\pi)$. Hence the Nyquist rate of $y(t)$ is 2000π and the maximum sampling time to allow perfect reconstruction is $T = 2\pi/(2000\pi) = 1/1000$.

b. Let $y(t) = 2\pi x_1(t)x_2(t)$. Then $\mathcal{F}\{y\} = \mathcal{F}\{x_1\} * \mathcal{F}\{x_2\}$ which is bandlimited in the frequency interval $(-1000\pi - 2000\pi, 1000\pi + 2000\pi)$. Hence the Nyquist rate of $y(t)$ is 6000π and the maximum sampling time to allow perfect reconstruction is $T = 2\pi/(6000\pi) = 1/3000$.

EEE 303 HW # 7 SOLUTIONS

Problem 8.4

Since $v(t) = g(t)\sin 400\pi t$ is passed through an ideal lowpass filter and this operation is best described in the frequency domain, we need to compute $V(jw)$. This can be achieved by either trigonometric identities (for this specific problem) or by performing the various frequency-domain convolutions.

Let $w_0 = 200\pi$. Then,

$$\begin{aligned}
X(jw) &= \frac{\pi}{j}\delta(w-w_0) - \frac{\pi}{j}\delta(w+w_0) + \frac{2\pi}{j}\delta(w-2w_0) - \frac{2\pi}{j}\delta(w+2w_0) \\
G(jw) &= \frac{1}{2\pi}X(jw) * \left[\frac{\pi}{j}\delta(w-2w_0) - \frac{\pi}{j}\delta(w+2w_0)\right] \\
&= \frac{1}{2j}X(j(w-2w_0)) - \frac{1}{2j}X(j(w+2w_0)) \\
V(jw) &= \frac{1}{2\pi}G(jw) * \left[\frac{\pi}{j}\delta(w-2w_0) - \frac{\pi}{j}\delta(w+2w_0)\right] \\
&= \frac{1}{2j}G(j(w-2w_0)) - \frac{1}{2j}G(j(w+2w_0)) \\
&= \frac{1}{-4}X(j(w-4w_0)) - \frac{1}{-4}X(jw) \\
&\quad -\frac{1}{-4}X(jw) + \frac{1}{-4}X(j(w+4w_0)) \\
&= -\frac{1}{4}X(j(w-4w_0)) + \frac{1}{2}X(jw) - \frac{1}{4}X(j(w+4w_0)) \\
&= -\frac{1}{4}\left[\frac{\pi}{j}\delta(w-5w_0) - \frac{\pi}{j}\delta(w-3w_0) + \frac{2\pi}{j}\delta(w-6w_0) - \frac{2\pi}{j}\delta(w-2w_0)\right] \\
&\quad +\frac{1}{2}\left[\frac{\pi}{j}\delta(w-w_0) - \frac{\pi}{j}\delta(w+w_0) + \frac{2\pi}{j}\delta(w-2w_0) - \frac{2\pi}{j}\delta(w+2w_0)\right] \\
&\quad -\frac{1}{4}\left[\frac{\pi}{j}\delta(w+3w_0) - \frac{\pi}{j}\delta(w+5w_0) + \frac{2\pi}{j}\delta(w+2w_0) - \frac{2\pi}{j}\delta(w+6w_0)\right] \\
&= -\frac{\pi}{2j}\delta(w-6w_0) - \frac{\pi}{4j}\delta(w-5w_0) + \frac{\pi}{4j}\delta(w-3w_0) + \frac{3\pi}{2j}\delta(w-2w_0) + \frac{\pi}{2j}\delta(w-w_0) \\
&\quad -\frac{\pi}{2j}\delta(w+w_0) - \frac{3\pi}{2j}\delta(w+2w_0) - \frac{\pi}{4j}\delta(w+3w_0) + \frac{\pi}{4j}\delta(w+5w_0) + \frac{\pi}{2j}\delta(w+6w_0)
\end{aligned}$$

Filtering $v(t)$ with a lowpass filter with cutoff frequency $2w_0$ and passband gain of 2 we get an output, say $y(t)$, that has frequency content in $|w| \leq 2w_0$. This operation is ambiguous in that there is an impulse exactly at the cutoff frequency. Formally, the ideal lowpass filter will only allow half of that impulse to go through.

$$Y(jw) = H(jw)V(jw) = +\frac{3\pi}{2j}\delta(w-2w_0) + \frac{\pi}{j}\delta(w-w_0) - \frac{\pi}{j}\delta(w+w_0) - \frac{3\pi}{2j}\delta(w+2w_0)$$

Hence, converting back to time domain

$$y(t) = \mathcal{F}^{-1}\left\{Y(jw)\right\} = \sin 200\pi t + \frac{3}{2}\sin 400\pi t$$

Problem 8.34

Let us first define the intermediate signals $v(t) = x(t) + \cos w_c t$, $z(t) = v^2(t)$. Then

$$\begin{aligned}
V(jw) &= X(jw) + \pi\delta(w-w_c) + \pi\delta(w+w_c) \\
Z(jw) &= \frac{1}{2\pi}V(jw) * V(jw)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}X(j(w-w_c)) + \frac{\pi}{2}\delta(w-2w_c) + \frac{\pi}{2}\delta(w) \\
&\quad + \frac{1}{2}X(j(w+w_c)) + \frac{\pi}{2}\delta(w) + \frac{\pi}{2}\delta(w+2w_c) \\
&\quad + \frac{1}{2}X(j(w-w_c)) + \frac{1}{2}X(j(w+w_c)) + \frac{1}{2\pi}X(jw) * X(jw) \\
&= X(j(w-w_c)) + X(j(w+w_c)) + \frac{\pi}{2}\delta(w-2w_c) + \frac{\pi}{2}\delta(w+2w_c) + \pi\delta(w) + \frac{1}{2\pi}X(jw) * X(jw)
\end{aligned}
$$

The exact expression of $X(jw) * X(jw)$ is irrelevant here; we only need the fact that, since $X(jw)$ is bandlimited in $(-w_M, w_M)$, then $X(jw) * X(jw)$ is bandlimited in $(-2w_M, 2w_M)$.

The same result is obtained by squaring $v(t)$ in time-domain and then taking its Fourier transform

$$
\begin{aligned}
z(t) &= x^2(t) + \cos^2 w_c t + 2x(t)\cos w_c t \\
&= x^2(t) + \frac{1}{2} + \frac{1}{2}\cos 2w_c t + 2x(t)\cos w_c t \\
Z(jw) &= \frac{1}{2\pi}X(jw) * X(jw) + \pi\delta(w)\frac{\pi}{2}\delta(w-2w_c) + \frac{\pi}{2}\delta(w+2w_c) + X(j(w-w_c)) + X(j(w+w_c))
\end{aligned}
$$

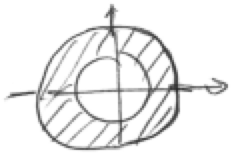
For $y(t) = x(t)\cos w_c t$, we need to have that $Y(jw) = H(jw)Z(jw) = \frac{1}{2}[X(j(w-w_c)) + X(j(w+w_c))]$ Hence, H should be a bandpass filter with a gain $\boxed{A = 1/2}$ and $\boxed{2w_M < w_l < w_c - W_M}$, $\boxed{w_c + w_M < w_h < 2w_c}$.

The feasibility constraints for this solution are $2w_M < w_c - W_M$, or $\boxed{w_c > 3w_M}$. (This also imples that $w_c + w_M < 2w_c$.)

EEE 303 HW Solutions <u>Ch 10</u>

<u>10.3</u> $x[n] = (-1)^n u[n] + \alpha^n u[-n-n_0]$

ROC : $1 < |z| < 2$



Determine α, n_0

a) $\mathcal{Z}\{x[n]\} = \sum_{-\infty}^{\infty} x[n] z^{-n}$

$$= \sum_{-\infty}^{\infty} (-1)^n u[n] z^{-n} + \sum_{-\infty}^{\infty} \alpha^n u[n-n_0] z^{-n}$$

1) $\sum (-1)^n u[n] z^{-n}$ converges iff $\left|\frac{-1}{z}\right| < 1 \Leftrightarrow |z| > 1$

2) $\sum \left(\frac{\alpha}{z}\right)^n u[-n-n_0]$ converges iff $\left|\frac{\alpha}{z}\right| > 1 \Leftrightarrow |z| < |\alpha|$

<u>Notice:</u> $\sum \left(\frac{\alpha}{z}\right)^n u[-n-n_0] = \sum_{-\infty}^{n_0} \left(\frac{\alpha}{z}\right)^n$

$$= \begin{cases} \sum_{-n_0}^{-1} \left(\frac{z}{\alpha}\right)^k + \sum_{0}^{\infty} \left(\frac{z}{\alpha}\right)^k & \text{if } n_0 > 0 \\ -\sum_{0}^{-n_0-1} \left(\frac{z}{\alpha}\right)^k + \sum_{0}^{\infty} \left(\frac{z}{\alpha}\right)^k & \text{if } n_0 < 0 \end{cases}$$

In both cases the first sum is finite and converges for all $\alpha \neq 0$, the second sum converges for $\left|\frac{z}{\alpha}\right| < 1$

Thus, for ROC : $1 < |z| < 2 \Rightarrow |\alpha| \geq 2$

($|\alpha| = 2$ for ROC to be exactly $1 < |z| < 2$)

n_0 does not affect ROC.

10.4 $x[n] = \frac{1}{3}^n \cos(\frac{\pi}{4}n)\, u(-n)$

$x[-n] = 3^n \cos \frac{\pi}{4} n \; u[n]$

$$\mathcal{Z}\{x(-n)\} \underset{10.2.11}{=} \frac{1 - 3\cos\frac{\pi}{4} z^{-1}}{1 - 2\cdot 3 \cdot \cos\frac{\pi}{4} z^{-1} + 3^2 z^{-2}} \quad ; \quad |z| > 3$$

$$\Rightarrow \mathcal{Z}\{x[n]\} \underset{10.5.4}{=} \frac{1 - 3\cos\frac{\pi}{4} z}{1 - 6\cos\frac{\pi}{4} z + 9z^2} \quad ; \quad |z| < \frac{1}{3}$$

We find the poles as roots of the denominator

$= 0.2357 \pm 0.2357j$

(their absolute value is $1/3$)

10.9 $$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})} \qquad |z| > 2$$

$$= \frac{A}{1-z^{-1}} + \frac{B}{1+2z^{-1}} \qquad |z|>2$$

PFE: $A = \left.\frac{1-\frac{1}{3}z^{-1}}{1+2z^{-1}}\right|_{z^{-1}=1} = 2/9$

$B = \left.\frac{1-\frac{1}{3}z^{-1}}{1-z^{-1}}\right|_{z^{-1}=-1/2} = 14/18$

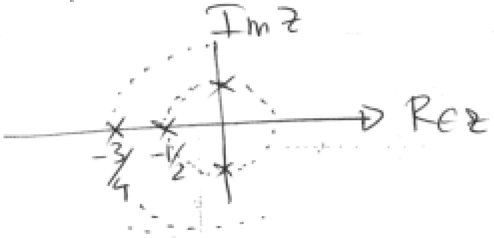
$$= \frac{2/9}{1-z^{-1}} + \frac{14/18}{1+2z^{-1}} \qquad |z|>2$$

$$\mathcal{Z}^{-1}\{X(z)\} = \underbrace{\frac{2}{9}\, u[n]}_{\alpha = +1} + \underbrace{\frac{14}{18}(-2)^n u[n]}_{\alpha = -2}$$

10.7 The ROC for the z-transform is annular regions of the form $a_1 < |z| < a_2$ and cannot contain poles. The poles of the given $X(z)$ are

$$\frac{1}{4}z^{-2} + 1 = 0 \Rightarrow z = \pm \tfrac{1}{2}j$$

$$\frac{3}{8}z^{-2} + \frac{5}{4}z^{-1} + 1 \Rightarrow z = -\tfrac{3}{4}, -\tfrac{1}{2}, \quad \text{i.e.,}$$

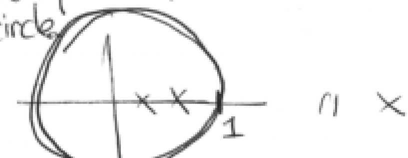


There are three possibilities for the ROC:

1. $|z| < \frac{1}{2}$ (left-sided sequence)
2. $\frac{1}{2} < |z| < \frac{3}{4}$ (two sided sequence; the poles with mag $\frac{1}{2}$ are inverted as right-sided and the pole with mag. $\frac{3}{4}$ are left sided)
3. $\frac{3}{4} < |z|$ (right-sided sequence).

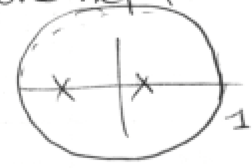
10.16 If the systems are stable, then the ROC must contain the unit circle. For causality, all poles must be inverted as right-sided, i.e. be inside the unit circle

a) the poles are at $\frac{1}{2}, \frac{1}{3}$ and ∞

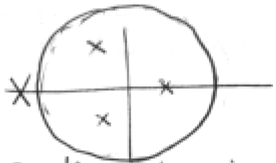


the pole at ∞ must be inverted as left-sided (anti causal). It is a one-step prediction.
$\therefore$ the system is non-causal

b) the poles are at $-0.75, 0.25$



Both are inverted as right-sided so the system is causal.



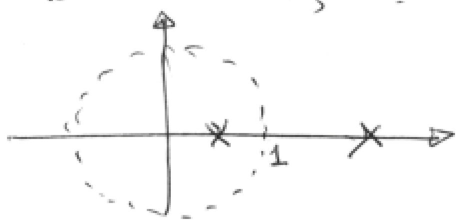
c). the poles are at -1.33, 0.794, $-0.397 \pm 0.687j$
the pole at -1.33 must be inverted as anti causal $\therefore$ the system is non-causal

10.35 Unit sample response : $\mathcal{Z}^{-1}\{G(z)\}$

$$G(z) = \frac{1}{z^{-1} - \frac{5}{2} + z}$$

(transfer function $= \dfrac{Y(z)}{X(z)}$ found "by inspection")

$$= \frac{z}{z^2 - \frac{5}{2}z + 1}$$; poles at 2, 0.5



3 cases of ROC:
1) $|z| < 1/2$
2) $1/2 < |z| < 2$
3) $2 < |z|$

Perform PFE : $G(z) = \dfrac{A}{z - 1/2} + \dfrac{B}{z-2} = \dfrac{-0.333}{z - 1/2} + \dfrac{1.333}{z-2}$

and invert each pole according to ROC: (see Table 10.2)

1). $\dfrac{1}{z-\alpha}$; $|z| > |\alpha| \leftrightarrow \alpha^{n-1} u[n-1]$

2). $\dfrac{1}{z-\alpha}$; $|z| < |\alpha| \leftrightarrow -\alpha^{n-1} u[-n]$

Thus,

1). $g[n] = \mathcal{Z}^{-1}\{G(z)\} = (-0.333)(-1)\left(\frac{1}{2}\right)^{n-1} u[-n] + (1.333)(-1)\, 2^{n-1} u[-n]$

$= 0.333 \frac{1}{2^{n-1}} u[-n] - 1.333\, 2^{n-1} u[-n]$

2) $g[n] = -0.333 \frac{1}{2^{n-1}} u[n-1] - 1.333\, 2^{n-1} u[-n]$

3) $g[n] = -0.333 \frac{1}{2^{n-1}} u[n-1] + 1.333\, 2^{n-1} u[n-1]$

EEE 303 HW # 8 SOLUTIONS

Problem 1.54

a. Let $S_N = \sum_{n=0}^{N-1} a^n$. Then, $aS_N = \sum_{n=1}^{N} a^n$. Therefore, $(1-a)S_N = a^0 - a^N$, from which we get

$$S_N = \sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

provided that $a \neq 1$. If $a = 1$, then simply $S_N = \sum_{n=0}^{N-1} 1^n = N$.

b. Let $S_\infty = \sum_{n=0}^{\infty} a^n = \lim_{N\to\infty} S_N$, provided that the limit exists. For this, the limit of a^N should exist as $N \to \infty$ which occurs if $|a| < 1$. In the complex case, $a^N = (|a|e^{j\theta})^N = |a|^N e^{j\theta N}$. So, if $|a| < 1$, $\lim_{N\to\infty} a^N = 0$, implying

$$S_\infty = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

c. Observe that for $|a| < 1$,

$$a\frac{dS_\infty}{da} = a\sum_{n=0}^{\infty} \frac{da^n}{da} = \sum_{n=0}^{\infty} na^n = a\frac{d}{da}\left(\frac{1}{1-a}\right) = \frac{1}{(1-a)^2}$$

Notice that the exchange of summation and differentiation makes sense since the series are absolutely convergent.

d. Again for $|a| < 1$,

$$\sum_{n=k}^{\infty} a^n = \sum_{n=0}^{\infty} a^n - \sum_{n=0}^{k-1} a^n = \frac{1}{1-a} - \frac{1-a^k}{1-a} = \frac{a^k}{1-a}$$

Problem 2.21

a. Let $x(n) = a^n U(n)$, $h(n) = b^n U(n)$. Then

$$\begin{aligned}
y(n) &= (h*x)(n) = \sum_{k=-\infty}^{\infty} b^{n-k}U(n-k)a^k U(k) \\
&= \sum_{k=0}^{n} b^{n-k}a^k \\
&= b^n \sum_{k=0}^{n}\left(\frac{a}{b}\right)^k, \quad \text{(assuming } b \neq a\text{)} \\
&= b^n \frac{(a/b)^{n+1}-1}{(a/b)-1} \\
&= \frac{a^{n+1}-b^{n+1}}{a-b}
\end{aligned}$$

Obviously, if $a = b$, then $y(n) = (n+1)b^n$.

b. Let $x(n) = U(n) - U(n-5)$, $h(n) = h_0(n-2) + h_0(n-11)$, where $h_0(n) = U(n) - U(n-6)$. Then, $x(n) * h(n) = y_0(n-2) + y_0(n-11)$, where $y_0(n) = x(n) * h_0(n)$. The last sequence is easy to compute:

n	-1	0	1	2	3	4	5	6	7	8	9	10	11
$y_0(n)$	0	1	2	3	4	5	5	4	3	2	1	0	0

From this, we get

n	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	...
$y_0(n-2)$	0	0	0	1	2	3	4	5	5	4	3	2	1	0	0	0	0	0	...
$y_0(n-11)$	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	5	...
$x(n) * h(n)$	0	0	0	1	2	3	4	5	5	4	3	2	2	2	3	4	5	5	...

Problem 2.31

Consider the difference equation

$$y(n) + 2y(n-1) = x(n) + 2x(n-2)$$

with $x(n) = \{\ldots 0, 1, 2, 3, 2, 2, 1, 0, \ldots\}$ for $n = \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$, respectively.

Assuming that $y(n)$ is "at rest" at $-\infty$, the above recursion $(y(n) = -2y(n-1) + x(n) + 2x(n-2))$ yields:

$$\begin{aligned}
& \vdots && (y(n) = 0, \text{ for } n < -3) \\
y(-3) &= && -2y(-4) + x(-3) + 2x(-5) = 0 \\
y(-2) &= && -2y(-3) + x(-2) + 2x(-4) = 1 \\
y(-1) &= && -2y(-2) + x(-1) + 2x(-3) = 0 \\
y(0) &= && -2y(-1) + x(0) + 2x(-2) = 5 \\
y(1) &= && -2y(0) + x(1) + 2x(-1) = -4 \\
y(2) &= && -2y(1) + x(2) + 2x(0) = 16 \\
y(3) &= && -2y(2) + x(3) + 2x(1) = -27 \\
y(4) &= && -2y(3) + x(4) + 2x(2) = 58 \\
y(5) &= && -2y(4) + x(5) + 2x(3) = -114 \\
y(6) &= && -2y(5) + x(6) + 2x(4) = 2 * 114 \\
y(7) &= && -2y(6) + x(7) + 2x(5) = -4 * 114 \\
y(8) &= && -2y(7) + x(8) + 2x(6) = 8 * 114 \\
& \vdots && (y(n) = -114(-2)^{n-5}, \text{ for } n \geq 5)
\end{aligned}$$

Problem 2.55 a (extra) $y[0] = x[0] = 1$. $h[n]$ satisfies the equation

$$h[n] = \frac{1}{2}h[n-1], \quad n \geq 1$$

with axuliary condition $h[0] = 1$. The characteristic polynomial is given by:

$$z - \frac{1}{2} = 0$$

then the solution for the homogeneous difference equation is

$$\begin{aligned}
h[n] &= C_0 \left(\frac{1}{2}\right)^n U[n] \\
h[0] &= 1
\end{aligned}$$

so $C_0 = 1$ then

$$h[n] = \left(\frac{1}{2}\right)^n U[n]$$

#1.37

$$\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)\, y(\tau)\, d\tau$$

a. $$\phi_{xy}(t) \underset{\tau' = t+\tau}{=} \int_{-\infty}^{\infty} x(\tau')\, y(-t+\tau')\, d\tau' = \boxed{\phi_{yx}(-t)}$$

Reflection, i.e., $\phi_{xy} = R[\phi_{yx}]$

b. $$\phi_{xx,\,odd}(t) = \frac{1}{2}\left(\phi_{xx}(t) - \phi_{xx}(-t)\right) =$$

$$= \frac{1}{2}\left(\int_{-\infty}^{\infty} x(t+\tau)\, x(\tau)\, d\tau - \int_{-\infty}^{\infty} x(-t+\tau)\, x(\tau)\, d\tau\right)$$

$\leftarrow \tau' = -t+\tau \rightarrow$

$$= \frac{1}{2}\left(\int_{-\infty}^{\infty} x(t+\tau)\, x(\tau)\, d\tau - \int_{-\infty}^{\infty} x(\tau')\, x(t+\tau')\, d\tau'\right)$$

$$= \boxed{0}$$ (Autocorrelation is even)

c. $y(t) = x(t+T)$

$$\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)\, y(\tau)\, d\tau = \int_{-\infty}^{\infty} x(t+\tau)\, x(\tau+T)\, d\tau$$

$$\underset{\tau' = \tau+T}{=} \int_{-\infty}^{\infty} x(t+\tau'-T)\, x(\tau')\, d\tau' = \boxed{\phi_{xx}(t-T)}$$

$$\phi_{yy}(t) = \int_{-\infty}^{\infty} y(t+\tau)\, y(\tau)\, d\tau = \int_{-\infty}^{\infty} x(t+\tau+T)\, x(\tau+T)\, d\tau$$

$$\underset{\tau' = \tau+T}{=} \int_{-\infty}^{\infty} x(\tau'+t)\, x(\tau')\, d\tau' = \boxed{\phi_{xx}(t)}$$