EEE 304, Test 1 **NAME: _SOLUTIONS_____**

Closed book-closed notes, transform tables allowed, 30'

Problem 1:

Consider the continuous time system with transfer function $H(s) = \frac{1}{s^2 - 2}$.

1. Can it be causal? If yes, sketch the ROC.

   It is causal for the ROC $\{s: 2 < Re\ s\}$

2. Can it be stable? If yes, sketch the ROC.

   It is stable for the ROC $\{s: -2 < Re\ s < 2\}$

3. Compute the step response (x(t) = u(t)) of the causal system.

$$y(t) = L^{-1}\left\{\frac{1}{(s+\sqrt{2})(s-\sqrt{2})s}\right\}_{ROC:\{2<Re\ s\}} = L^{-1}\left\{\frac{1/4}{(s+\sqrt{2})} + \frac{1/4}{(s-\sqrt{2})} + \frac{-1/2}{s}\right\}_{R-S} \ldots$$

$$y(t) = \left\{\frac{1}{4}e^{-\sqrt{2}t} + \frac{1}{4}e^{\sqrt{2}t} - \frac{1}{2}\right\}u(t)$$

Problem 2:

For the stable discrete time system with transfer function $H(z) = \frac{z-1}{z(z^2 - 0.16)}$

1. Sketch the region of convergence of H(z). Is the system causal?

   The system is stable for the ROC $\{z: \sqrt{0.16} = 0.4 < |z|\}$, which includes the unit circle. Since the ROC extends to ∞, the system is also causal.

2. Compute the unit step response (x(n)=u(n)).

$$y(n) = Z^{-1}\left\{\frac{z-1}{z(z+0.4)(z-0.4)} \cdot \frac{z}{z-1}\right\}_{ROC:\{1<|z|\}} = Z^{-1}\left\{\frac{-1/0.8}{(z+0.4)} + \frac{1/0.8}{(z-0.4)}\right\}_{R-S} \ldots$$

$$y(n) = \{1.25(0.4)^{n-1} - 1.25(-0.4)^{n-1}\}u(n-1)$$