EEE 304, Test 3 **NAME:_____SOLUTIONS______**

45min, Closed-book, closed-notes, calculators and transform tables allowed

Problem 1:

Suppose that a continuous time signal x(t) is approximately bandlimited and it is processed by a CT system with transfer function $H(s) = \frac{1}{s+1}$. Design a DT system with ideal sampling and reconstruction to replace the CT system. More specifically,

1. Select the parameters of an Anti-Aliasing Filter, if necessary.
2. Select a suitable sampling rate.
3. Provide the design equation for the DT filter transfer function, which is to be approximated in the implementation.

In all cases, provide a short justification for your answer.

Without any additional information, the low-pass filter H(s) attenuates the bandlimited signal X(jw) for frequencies greater than 1rad/s, so a maximum frequency of 10rad/s should be adequate to describe most of the output signal. (Within -20dB; the interval can be increased when more accuracy is desired.)

We now select the AAF filter to have bandwidth 10rad/s (1.6Hz), from which it follows that the sampling rate should be at least 3.2Hz.

To enable exact reconstruction of frequencies up to 1.6Hz with an ideal low-pass filter, the design equation that the discrete filter should satisfy becomes

$$H_d\left(e^{j\Omega}\right) = H_d\left(e^{jwT}\right) = H(jw) = \frac{1}{jw+1}$$

We can also write the expression

$$H_d(z) = \frac{1}{\frac{\ln z}{T} + 1}$$

For implementation, this should be approximated by a finite-dimensional transfer function, e.g., via Tustin, Euler, etc.

Problem 2:

Find the largest sampling interval T_s to allow perfect reconstruction of the signals

1. $\cos(t)u(t)$
2. $y = H[x], \quad where: x(t) = \sin(t), H(s) = \frac{1}{s+1}$
3. $\sin^N 4t, \quad where\ N > 1$

1. $w_{max1} = 1, w_{max2} = \infty \Rightarrow w_{max} = 1 + \infty = \infty \Rightarrow T_s = 0$
2. $y = h * x, w_{max1} = \infty, w_{max2} = 1 \Rightarrow w_{max} = \min(1, \infty) = 1 \Rightarrow T_s = \frac{\pi}{1}$
3. $w_{max1} = 4, \ldots, w_{maxN} = 4 \Rightarrow w_{max} = 4 + \cdots + 4 = 4N \Rightarrow T_s = \frac{\pi}{4N}$

Problem 3:
Define the terms: Anti-Aliasing filter, Nyquist frequency, Nyquist rate.

“Anti-Aliasing filter” is a low-pass, continuous-time filter which is used to process the signal before sampling in order to eliminate any high frequencies that can cause aliasing; the filter bandwidth should be at most half the sampling frequency.

“Nyquist frequency,” for a sampling system, is the highest frequency that can be reconstructed with an ideal low-pass filter $(f_s/2)$.

“Nyquist rate,” for a signal, is the lowest sampling frequency that can be used to sample and reconstruct the signal without aliasing $(2f_{max})$.