**EEE 304, TEST 4**   NAME:___SOLUTIONS__

Closed-book, closed-notes. Transform Tables and calculators allowed, 30'

Problem:

Determine the signal produced if the following sequence of operations is performed on a signal $x(t)$ that is bandlimited to w_m (i.e., $X(jw) = 0$ for $|w| > w_m$). Suppose that $X(jw)$ is as shown in the figure and has amplitude 1. What is the amplitude of the output signal relative to $X(jw)$?

1. Modulation with a square wave with period $T = \pi/2w_m$ and 25% duty cycle, i.e., $s(t) = 1$ for $|t| < T/8$.
2. Bandpass filtering with an ideal filter $H(jw) = 1$ for $4w_m < |w| < 5w_m$.
3. Modulation with $\cos(4w_m t)$.
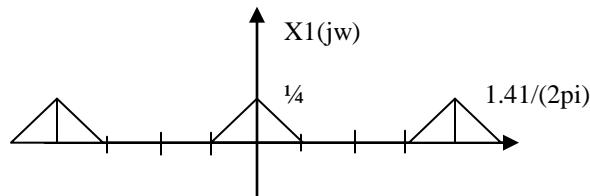4. Lowpass filtering with an ideal filter $H(jw) = 1$ for $|w| < 3w_m$.

X(jw)
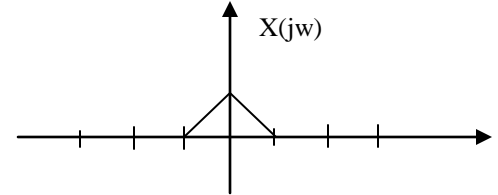
We have,

$$S(jw) = \sum 2\pi a_k \delta(w - w_0 k) = \sum \frac{2\pi(\sin kw_0 T_1)}{k\pi}\, \delta(w - w_0 k) = \sum \frac{2\left(\sin\frac{k\pi}{4}\right)}{k}\, \delta(w - w_0 k)$$

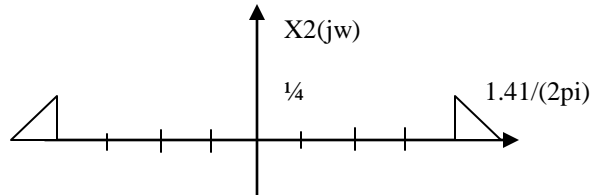$$w_0 = \frac{2\pi}{T} = 4w_m, \quad T_1 = \frac{\pi}{16w_m}$$

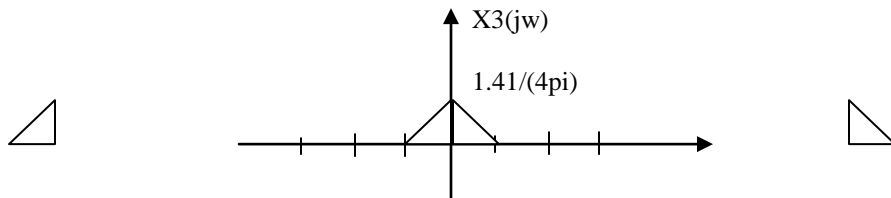$$S(jw) = \frac{\pi}{2}\delta(w) + \sqrt{2}\delta(w - 4w_m) + 1\delta(w - 8w_m) + \cdots$$

Then

1. $X_1(jw) = \frac{1}{2\pi} X(jw) * S(jw)$

X1(jw)
¼   1.41/(2pi)

2. $X_2(jw) = X_1(jw) * H_1(jw)$

X2(jw)
¼   1.41/(2pi)

3. $X_3(jw) = \frac{1}{2\pi} X_2(jw) * C(jw) = \frac{1}{2} X_2(jw) * [\delta(w - 4w_m) + \delta(w + 4w_m)]$

X3(jw)
1.41/(4pi)

4. $X_4(jw) = X_3(jw) * H_3(jw)$

X4(jw)
1.41/(4pi)