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EEE 304 **HW 1**

Problem 1:

Consider the filter with impulse response $h(t) = e^{-t}u(t-1) - e^{-2t}u(t)$.

1. Find the transfer function

2. Find the Laplace transform of the output when $x(t) = \sin(t)u(t)$

3. Find the output by taking the inverse Laplace transform of your answer to part 2.

4. Can you obtain the same result using Fourier Transforms?

1. $H(s) = \frac{1}{e}L\{e^{-(t-1)}u(t-1)\} - L\{e^{-2t}u(t)\} = \frac{1}{e}\frac{e^{-s}}{s+1} - \frac{1}{s+2}$

2. $y(s) = \left(\frac{1}{e}\frac{e^{-s}}{s+1} - \frac{1}{s+2}\right)\frac{1}{s^2+1}$

3. $y(t) = L^{-1}\left\{\frac{1}{e}\frac{e^{-s}}{s+1}\frac{1}{s^2+1}\right\} - L^{-1}\left\{\frac{1}{s+2}\frac{1}{s^2+1}\right\} = \frac{1}{e}L^{-1}\left\{\frac{A}{s+1} + \frac{B}{s-j} + \frac{B^*}{s+j}\right\}_{\{t=t-1\}} - L^{-1}\left\{\frac{C}{s+2} + \frac{D}{s-j} + \frac{D^*}{s+j}\right\}$

$= \frac{1}{e}L^{-1}\left\{\frac{1/2}{s+1} + \frac{1/2j(j+1)}{s-j} + \frac{B^*}{s+j}\right\}_{\{t=t-1\}} - L^{-1}\left\{\frac{1/5}{s+2} + \frac{1/2j(2+j)}{s-j} + \frac{D^*}{s+j}\right\}$

$= \frac{1}{e}\left\{\frac{1}{2}e^{-t}u(t) + 2Re\frac{1}{2j(j+1)}e^{jt}\right\}_{\{t=t-1\}} - \left\{\frac{1}{5}e^{-2t}u(t) + 2Re\frac{1}{2j(2+j)}e^{jt}\right\}$

$= \frac{1}{e}\left\{\frac{1}{2}e^{-t}u(t) + Re\left|\frac{1}{j(j+1)}\right|e^{j\angle\frac{1}{j(j+1)}}e^{jt}u(t)\right\}_{\{t=t-1\}} - \left\{\frac{1}{5}e^{-2t}u(t) + Re\left|\frac{1}{j(2+j)}\right|e^{\angle\frac{1}{j(2+j)}}e^{jt}u(t)\right\}$

$= \frac{1}{2e}e^{-(t-1)}u(t-1) + \frac{1}{\sqrt{2}e}\cos\left(t-1-\frac{\pi}{2}-\tan^{-1}1\right)u(t-1) - \frac{1}{5}e^{-2t}u(t) - \frac{1}{\sqrt{5}}\cos\left(t-\frac{\pi}{2}-\tan^{-1}\frac{1}{2}\right)$

$= \frac{1}{2e}e^{-(t-1)}u(t-1) + \frac{1}{\sqrt{2}e}\sin(t-1-\tan^{-1}1)u(t-1) - \frac{1}{5}e^{-2t}u(t) - \frac{1}{\sqrt{5}}\sin\left(t-\tan^{-1}\frac{1}{2}\right)$ (other equivalent expressions are possible)

For verification, the following MATALB commands can be used:

```
>> H1=tf(1,[1 1])/exp(1)

>> H1.iodelay=1

>> H2=tf(-1,[1 2])

>> t=[0:.01:20]'; x=sin(t);

>> y=lsim(H1,x,t)+lsim(H2,x,t);

>> ut=ones(size(t)); ind=find(t<1); ut(ind)=ut(ind)*0;

>> yy=1/2/exp(1)*exp(-t+1).*ut+1/sqrt(2)/exp(1)*sin(t-1-atan(1)).*ut-1/5*exp(-2*t)-1/sqrt(5)*sin(t-atan(.5));

>> plot(t,y,t,yy)
```

4. The system is stable, hence the Fourier transform of its transfer function exists. The Fourier transform of the input also exists (in the sense of distributions) and so does the Fourier transform of the output. Hence, it is possible to obtain the same result but due to the transients the computation is much more involved (see also sample problem solutions).

Problem 2:

Consider the continuous time causal filter with transfer function

$$H(s) = \frac{s}{(s-1)(s-2)}$$

1. Compute the response of the filter to $x(t) = u(t)$.

2. Compute the response of the filter to $x(t) = u(-t)$.

3. Repeat parts 1 and 2 for a stable system with the same transfer function.

1. $y(s) = \frac{s}{(s-1)(s-2)}\frac{1}{s}; ROC = \{Re\ s > 2\} \cap \{Re\ s > 0\}$ (right-sided poles)

   $y(s) = \frac{-1}{(s-1)} + \frac{1}{(s-2)} => y(t) = -e^{t}u(t) + e^{2t}u(t)$

2. $ROC = \{Re\ s > 2\} \cap \{Re\ s < 0\} = \phi$ (no intersection, response is not well defined)

3.1 $y(s) = \frac{s}{(s-1)(s-2)}\frac{1}{s}; ROC = \{Re\ s < 1\} \cap \{Re\ s > 0\}$ (left-sided poles for H, ROC includes jw-axis)

   $y(s) = \frac{-1}{(s-1)} + \frac{1}{(s-2)} => y(t) = +e^{t}u(-t) - e^{2t}u(-t)$

3.2 $y(s) = \frac{s}{(s-1)(s-2)}\frac{-1}{s}; ROC = \{Re\ s < 1\} \cap \{Re\ s < 0\}$ (left-sided poles for H, ROC includes jw-axis)

   $y(s) = \frac{+1}{(s-1)} + \frac{-1}{(s-2)} => y(t) = -e^{t}u(-t) + e^{2t}u(-t)$

Problem 3:

Consider the discrete time stable filter with transfer function

$$H(z) = \frac{z}{(z-0.1)(z-0.2)}$$

1. Compute the response of the filter to $x[n] = u[n]$.

2. Repeat part 1 for a causal filter with the same transfer function.

1. $y(z) = \frac{z}{(z-0.1)(z-0.2)}\frac{z}{z-1}; ROC = \{|z| > 0.2\} \cap \{|z| > 1\}$ (right-sided poles for H, ROC includes unit circle)

   $y(z) = z\left\{\frac{-2.5}{(z-0.2)} + \frac{1.1}{(z-0.1)} + \frac{1.3889}{z-1}\right\} => y(n) = -2.5(0.2)^{n}u(n) + 1.1(0.1)^{n}u(n) + 1.3889u(n)$ (other equivalent expressions are also possible)

2. $ROC = \{|z| > 0.2\}$ also corresponds to the causal filter ROC, so the response is the same as in Part 1.