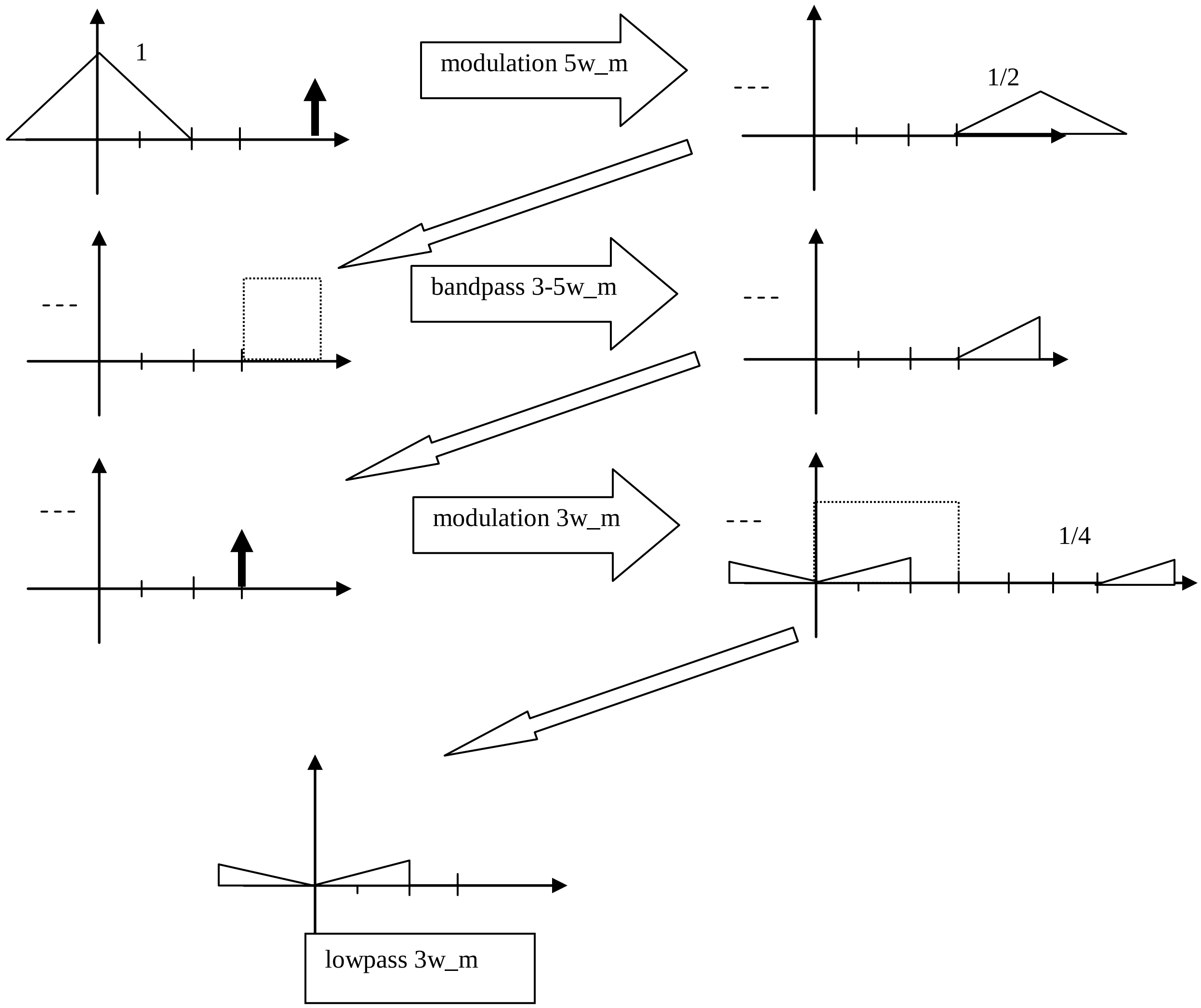
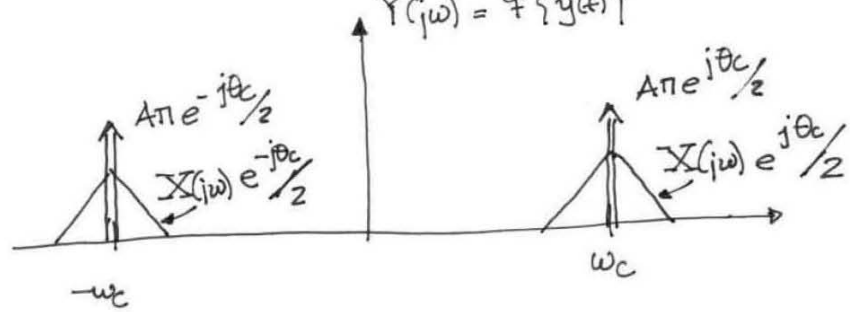
EEE 304, HW 5 SOLUTIONS

Problem 1:
Do Problem 8.22 from the textbook.

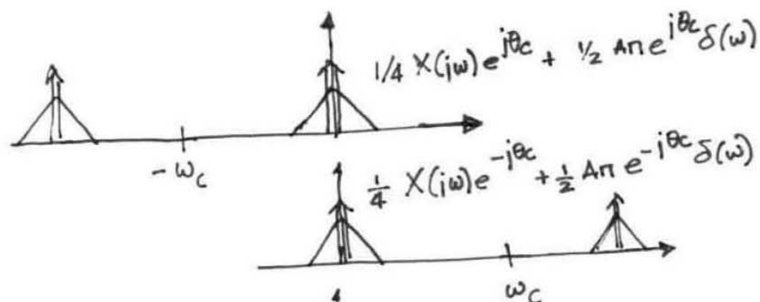


Problem 2:
Do Problem 8.26 from the textbook.

Pr 8.26



$\mathcal{F}\{y(t)\cos\omega_c t\}$ =



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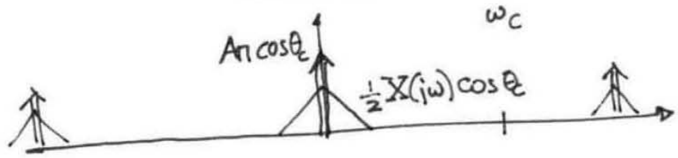
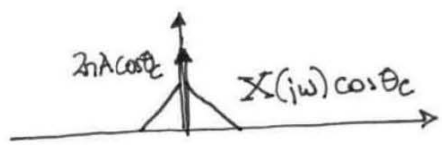


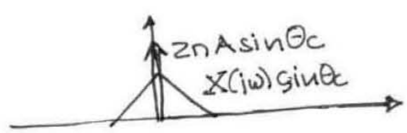
After lowpass $\longrightarrow$



$= \mathcal{F}\{(x(t)+A)\cos\theta_c\}$

Similarly for the $\sin\omega_c t$ modulation, (the side lobes are different)

$\longrightarrow \cdots \xrightarrow{\text{after lowpass}}$



$= \mathcal{F}\{(x(t)+A)\sin\theta_c\}$

Hence, the outputs of the low-pass filters are $(x(t)+A)\cos\theta_c$ and $(x(t)+A)\sin\theta_c$. Squaring and adding we obtain $[x(t)+A]^2[\cos^2\theta_c + \sin^2\theta_c]$ $= [x(t)+A]^2$. Therefore, $r(t) = x(t) + A$. (Note: $x(t)+A > 0$)

Problem 3: (8.49 of textbook)

(a) s(t) periodic so its Fourier transform is computed through the Fourier series expansion. That is,

$$s(t) = \sum a_k e^{jk\frac{2\pi}{T}t}, \quad \omega_o = \frac{2\pi}{T}$$

$$S(j\omega) = 2\pi \sum a_k \delta\left(\omega - k\frac{2\pi}{T}\right), \quad a_k = \frac{\sin k\omega_o T_1}{k\pi}, \quad T_1 = \frac{T}{4} \quad \Rightarrow a_k = \frac{\sin k\pi/2}{k\pi}$$

$$a_0 = 1/2$$

(The a_k is zero for even k, other than 0.)

The modulated (chopped) x(t) has Fourier transform

$$X_s(j\omega) = \frac{1}{2\pi} X(j\omega) * S(j\omega) = \sum a_k X\left(j\omega - jk\frac{2\pi}{T}\right)$$

yielding the band-pass filtered-chopped signal, say v(t)

$$V(j\omega) = H_1(j\omega) \sum a_k X\left(j\omega - jk\frac{2\pi}{T}\right) = Aa_1 X\left(j\omega - j\frac{2\pi}{T}\right) + Aa_{-1} X\left(j\omega + j\frac{2\pi}{T}\right)$$

$$= \frac{A}{\pi} X\left(j\omega - j\frac{2\pi}{T}\right) + \frac{A}{\pi} X\left(j\omega + j\frac{2\pi}{T}\right)$$

The maximum allowable frequency content in x(t) for this expression to be valid is $\omega_M < \pi/T$.

Next, V(jω) is re-modulated (chopped) and low-pass filtered. The modulation by s(t) produces two replicas of X(jω) at 0, each multiplied by $1/\pi$, yielding a total coefficient of $2A/\pi^2$. The rest of the replicas are at $k2\pi/T$ that are filtered out by the H_2 low-pass filter. So,

$$Y(j\omega) = H_2(j\omega) V_s(j\omega) = \frac{2A}{\pi^2} X(j\omega)$$

with the same condition on the maximum frequency in x(t), $\omega_M < \pi/T$.

(b) From the last expression, the equivalent gain of the overall system is $2A/\pi^2$.