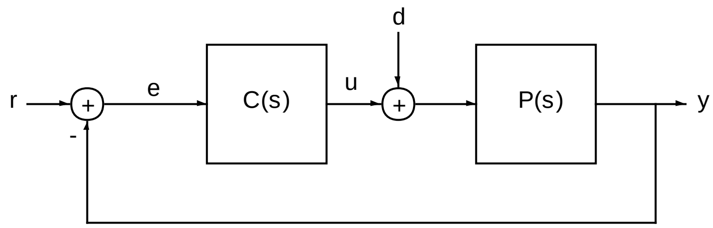
EEE 304, HW 5

Problem 1:
For the feedback system shown below, compute the transfer functions u/r, u/d, y/r, y/d.



$$\frac{u(s)}{r(s)} = \frac{C}{1+CP}, \quad \frac{u(s)}{d(s)} = \frac{-CP}{1+CP}, \quad \frac{y(s)}{r(s)} = \frac{PC}{1+PC}, \quad \frac{y(s)}{d(s)} = \frac{P}{1+PC}$$

Problem 2: (Low Bandwidth Controller)
For the feedback system of Problem 1, suppose $P(s) = 10/(0.3s+1)$.

a. When *C(s) = K*, design K so that the loop crossover frequency (i.e., w: |P(jw)C(jw) |= 1) is 0.8rad/s. What is the contribution of a constant unit disturbance to the output?
b. When $C(s) = K(Ts+1)/s$, design K,T so that the crossover frequency is 0.8rad/s and the phase margin (i.e., the difference between the loop angle and –180 at the crossover frequency, $\angle P(jw_c)C(jw_c)+180$) is at least 60°. What is the contribution of a constant unit disturbance to the output?
Verify in MATLAB, using **step(feedback(P,K),feedback(P,C))**

a. $\omega_c = 0.8 : |P(j\omega_c)C(j\omega_c)| = 1 \Rightarrow \left| \frac{10K}{\sqrt{1+(0.3\omega_c)^2}} \right| = 1 \Rightarrow |K| = 0.103$

$K > 0$ for stability $(\angle \mathrm{C} + \angle \mathrm{P} > -180)$

$$y_d(s) = \frac{P}{1+PC} d(s) = \frac{10}{0.3s+10K+1} \frac{1}{s} = \frac{-4.93}{s+6.77} + \frac{4.93}{s} \Rightarrow y_d(t) = 4.93\left(1-e^{-6.77t}\right)U(t)$$

$$\Rightarrow y_{d,ss} = \lim_{t\to\infty} y_d(t) = \left.\frac{P}{1+PC}\right|_{s=0} = 4.93$$

b. $\omega_c = 0.8 : \angle(1/s) + \angle \mathrm{P} = \text{-}103.5° > -120°$

$\Rightarrow T = 0$ (no zero is needed, the controller is an integrator)

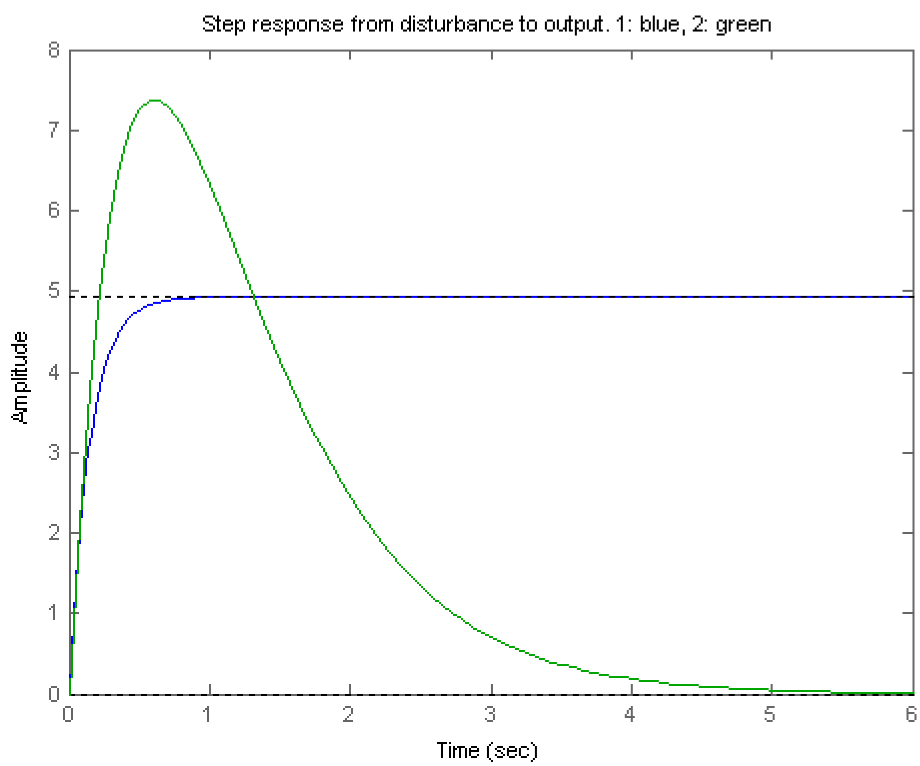
$$\omega_c = 0.8 : |P(j\omega_c)C(j\omega_c)| = 1 \Rightarrow \left|\frac{10K}{\omega_c\sqrt{1+(0.3\omega_c)^2}}\right| = 1 \Rightarrow |K| = 0.082$$

$K > 0$ for stability ($\angle \mathrm{C} + \angle \mathrm{P} > \text{-}180$)

$$\mathrm{C} = \frac{0.082}{\mathrm{s}}$$

$$y_d(s) = \frac{P}{1+PC}d(s) = \frac{10s}{0.3s^2 + s + 10K}\frac{1}{s} = \frac{10}{0.3s^2 + s + 0.82} = \frac{-7.29}{s\ +1.85} + \frac{7.29}{s\ +1.48}$$

$$\Rightarrow y_d(t) = \left\{-7.29e^{-1.85t} + 7.29e^{-1.48t}\right\}U(t) \Rightarrow y_{d,ss} = \lim_{t\to\infty} y_d(t) = \left.\frac{P}{1+PC}\right|_{s=0} = 0$$



Problem 3: (High Bandwidth Controller)

For the feedback system of Problem 1, suppose $P(s) = 1/(0.3s+1)$.

a. When $C(s) = K$, design K so that the loop crossover frequency (i.e., w: |P(jw)C(jw) |= 1) is 18rad/s. What is the contribution of a constant unit disturbance to the output?

b. When $C(s) = K(Ts+1)/s$, design K,T so that the crossover frequency is 18rad/s and the phase margin (i.e., the difference between the loop angle and –180 at the crossover frequency, $\angle P(jw_c)C(jw_c) + 180$) is at least 60°. What is the contribution of a constant unit disturbance to the output?
Verify in MATLAB, using **step(feedback(P,K),feedback(P,C))**

a. $\omega_c = 18: |P(j\omega_c)C(j\omega_c)| = 1 \Rightarrow \left|\frac{K}{\sqrt{1+(0.3*18)^2}}\right| = 1 \Rightarrow |K| = 5.49$

$K > 0$ for stability ($\angle C + \angle P > -180$)

$$y_d(s) = \frac{P}{1+PC}d(s) = \frac{1}{0.3s+6.49}\frac{1}{s} = \frac{-0.154}{s+2.16} + \frac{0.154}{s} \Rightarrow y_d(t) = 0.154\left(1-e^{-2.16t}\right)U(t)$$

$$\Rightarrow y_{d,ss} = \lim_{t\to\infty} y_d(t) = \left.\frac{P}{1+PC}\right|_{s=0} = 0.154$$

b. $\omega_c = 18: \angle C + \angle P = -170° + \angle(Ts+1) = -180 + PM = -120°$

$\Rightarrow T = \tan(50^o)/18 = 0.066$

$$\omega_c = 1: |P(j\omega_c)C(j\omega_c)| = 1 \Rightarrow \left|\frac{K(\sqrt{1+(0.066\omega_c)^2}}{\omega_c\sqrt{1+(0.3\omega_c)^2}}\right| = 1 \Rightarrow |K| = 1/0.016 = 63$$

$K > 0$ for stability ($\angle C + \angle P > -180$)

$$C = \frac{4.2s+63}{s}$$

$$y_d(s) = \frac{P}{1+PC}d(s) = \frac{s}{0.3s^2+5.2s+63}\frac{1}{s} = \frac{-0.14j}{s\ +8.67-11.7j} + \frac{0.14j}{s\ +8.67+11.7j}$$

$$\Rightarrow y_d(t) = 2\,\mathrm{Re}\left\{-0.14je^{-8.67t}e^{11.7jt}\right\}U(t) \Rightarrow y_{d,ss} = \lim_{t\to\infty} y_d(t) = \left.\frac{P}{1+PC}\right|_{s=0} = 0$$