**EEE 481 Test 2** **NAME: __SOLUTIONS___**

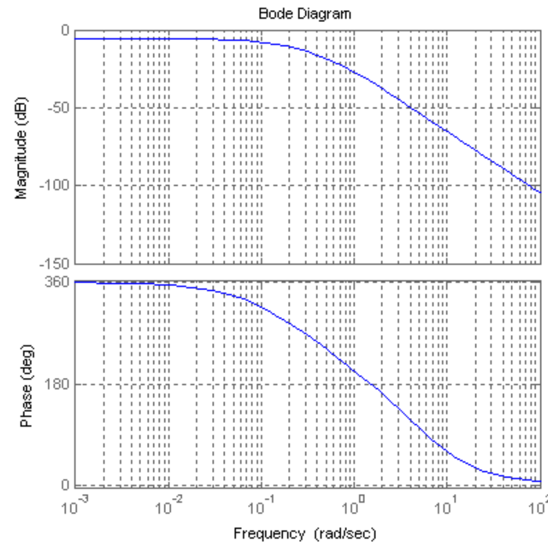
**75', Closed-book, Closed-Notes, Calculators and One 8 1/5 x 11 sheet (2pages) of notes and formulae allowed.**

1. Design a PID controller to achieve a crossover of 1 rad/s, 60deg phase margin, for the plant with transfer function (also shown in the adjacent plot)

$$P(s) = \frac{2(-0.2s+1)}{(7s+1)(s^2+6s+4)}$$

At 1rad/s, $\angle P = -156^o$, $|P| = 0.043$. The required phase lead from the compensator zero is 156+90-120 = 126$^o$, so we must use a PID. We choose a time constant for the derivative filter $\tau = 0.1$, so that its contribution is -6$^o$ phase lag. The total contribution from the PID zeros now becomes 132$^o$, or 66$^o$ per zero which is reasonable. The PID zeros are computed at -0.44 and the gain correction is 1/0.051. The final PID transfer function is $\frac{19.6\ s^2\ +17.3\ s+3.82}{0.1\ s^2+s}$

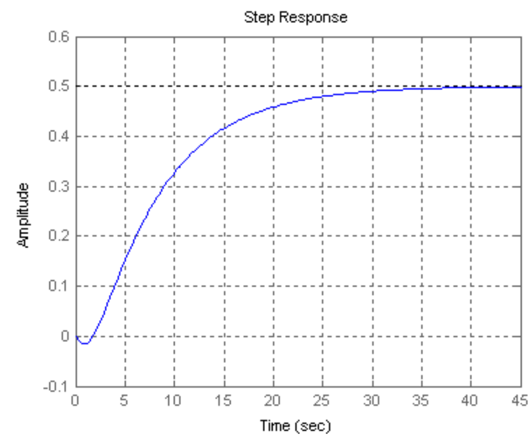
Another possibility is to use $\tau = 0.05$ for a -3$^o$ lag, anticipating the easier discretization with 0.1s sampling. Now the PID is $\frac{19.0\ s\text{^}2\ +18.0\ s+4.2}{0.05\ s^2+s}$.




2. Select a suitable sample time and discretize the controller. What will be the phase margin of the discrete time closed loop system?

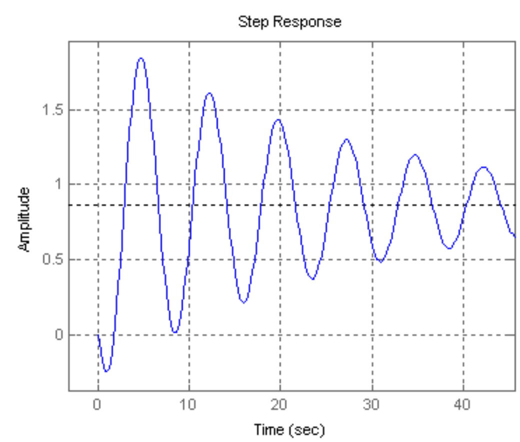
A $T_s = 0.1$sec sample time would contribute -3$^o$ phase lag (half-sample delay), which is reasonable. The Tustin discretization of the controller becomes $\frac{136.2\ z^2 - 260.6\ z + 124.7}{z^2 - 1.333\ z + 0.3333}$

For the alternative PID design, the Tustin discretization is $\frac{199.5\ z^2 - 380.7\ z + 181.6}{z^2 - z}$. The DT closed-loop PM is approximately 57$^o$.




3. Do you expect that a prefilter will be needed to avoid overshoots in the step response? Briefly describe the design of such a prefilter.

The zero is only at ½ the loop crossover so we expect only a moderate need for a prefilter. For its design, we would start with a lowpass filter (and its Tustin equivalent in DT) $F(s) = \frac{p}{z} \cdot \frac{s+z}{s+p}$, where $z \simeq BW, p \simeq 0.44$, and iterate the pole and the zero until a desired overshoot is obtained.




4. Use Ziegler-Nichols rules to design a PID for a system with the step response shown in the adjacent plot.

From the plot, we estimate R = 0.15/2.5 and L = 2.5, so

$$PID = \frac{8.3\ s^2 + 8\ s + 1.6}{s}$$

5. A system is tested in feedback with a proportional controller with gain 10. Use Ziegler-Nichols rules to design a PID controller if the closed-loop step response is as shown in the adjacent plot.

From the plot, we estimate Pu = 12-5 and Ku = 10, so

$$PID = \frac{5.3\ s^2 + 6.0\ s + 1.7}{s}$$

| *1* | P | PI | PID |
|---|---|---|---|
| **Kp** | 1/RL | 0.9/RL | 1.2/RL |
| **Ki** | - | 0.27/RL$^2$ | 0.6/RL$^2$ |
| **Kd** | - | - | 0.5/R |

| *1* | *P* | *PI* | *PID* |
|---|---|---|---|
| **Kp** | 0.5Ku | 0.45Ku | 0.6Ku |
| **Ki** | - | 0.54Ku/Pu | 1.2Ku/Pu |
| **Kd** | - | - | 0.075KuPu |