EEE 481, Test 1. **Name: ____SOLUTIONS_____**

75', Closed-book, Closed-Notes. Calculators and One 8 1/5 x 11 sheet (2pages) of notes and formulae allowed.

Problem 1:

Provide brief answers to the following questions:

1. Data from an A/D Converter are required to have a 1mV resolution in the range of [-10:10]V. What is the number of bits that should be used?
2. How long does it take to transmit ten 2-byte integers at 9600Baud?
3. How can we model the quantization error?

1. 1. $\frac{20}{2^N-1} = 1mV \Rightarrow N \geq \frac{\log(20000)}{\log(2)} = 14.28 \Rightarrow N = 15$

2. 2. $10 \times \frac{2(8+2)}{9600} = 0.021(s)$

3. 3. Quantization can be modeled as additive noise with uniform distribution of amplitude ½-LSB. It is fairly efficient for variance calculations.

Problem 2:

An continuous time system is has transfer function $H(s) = \frac{2}{0.1s+1}$. Using a sampling rate of 0.1 sec, determine the transfer functions of:

1. the discrete-time ZOH equivalent system.
2. the discrete-time system obtained by the forward Euler approximation of the derivative.
3. the discrete-time system obtained by the Tustin transformation.

1. $L^{-1}\left\{\frac{20}{s(s+10)}\right\}_{t=nT} = L^{-1}\left\{\frac{2}{s} - \frac{2}{s+10}\right\}_{t=nT} = \{2u(n) - 2e^{-10nT}u(n)\} \to Z\{y_s(nT)\} = \frac{2z}{z-1} - \frac{2z}{z-e^{-1}} \Rightarrow G_{ZOH}(z) = \frac{z-1}{z}Y_s(z) = 2 - 2\frac{z-1}{z-e^{-1}} = 2\frac{1-e^{-1}}{z-e^{-1}} = \frac{1.264}{z-0.368}$

2. $s = \frac{z-1}{T} \Rightarrow G_{FE}(z) = \frac{2}{0.1\frac{z-1}{T}+1} = \frac{2}{z}$

3. $s = \frac{2}{T}\cdot\frac{z-1}{z+1} \Rightarrow G_{TUS}(z) = \frac{2}{0.1\frac{2}{T}\frac{z-1}{z+1}+1} = \frac{2(z+1)}{2(z-1)+(z+1)} = \frac{2(z+1)}{3z-1}$

Problem 3:

Consider the discrete-time system with state space representation

$$x_{k+1} = Ax_k + Bu_k \qquad y_k = Cx_k \quad \text{where} \quad A = \begin{bmatrix} -0.1 & 0.2 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

1. Determine its transfer function.
2. Determine whether the system is controllable and observable (hence minimal), and stable.

1. $G(z) = C(zI - A)^{-1}B + D = \frac{2z+1}{z^2+0.1z-0.2} = \frac{2(s+0.5)}{(z+0.5)(z-0.4)}$

2. $Q_c = [B, AB]$**, has rank 2 so the system is controllable.** $Q_o = [C; CA]$ **has rank 1 so the system is not observable and hence not minimal. We can come to the same conclusion by observing that the transfer function has a common factor between numerator and denominator, so the system is not minimal (could be uncontrollable or unobservable or both). Finally, we compute the roots of the denominator (-0.5,0.4) which are both inside the unit circle, hence the system is stable.**