EEE 481, Test 1. **Name: ___SOLUTIONS___**

75', Closed-book, Closed-Notes. Calculators and One 8 1/5 x 11 sheet (2pages) of notes and formulae allowed.

Problem 1:

Provide brief answers to the following questions:

1. Data from a 10bit A/D Converter are required to have a 1mV resolution. What is the range of voltage that can be measured?
2. How many 2-byte integers can be transmitted in 1sec, at 19200Baud?
3. How can we model the quantization error?

1. ½-LSB resolution = $\frac{1}{2048}V_{max} = 1mV => V_{max} = 2.048(V)$.
2. $N \times \frac{2(8+2)}{19200} = 1(s) => N = \frac{19200}{20} = 960\ (numbers)$.
3. Quantization can be modeled as additive noise with uniform distribution of amplitude ½-LSB. It is fairly efficient for variance calculations.

Problem 2:

An continuous time system is has transfer function $3/(s+3)$. Using a sampling rate of 0.1 sec, determine the transfer functions of:

1. the discrete-time ZOH equivalent system.
2. the discrete-time system obtained by the forward Euler approximation of the derivative.
3. the discrete-time system obtained by the Tustin transformation.

1. $L^{-1}\left\{\frac{3}{s(s+3)}\right\}_{t=nT} = L^{-1}\left\{\frac{1}{s} - \frac{1}{s+3}\right\}_{t=nT} = \{u(n) - e^{-3nT}u(n)\} \rightarrow Z\{y_s(nT)\} = \frac{z}{z-1} - \frac{z}{z-e^{-0.3}} => G_{ZOH}(z) = \frac{z-1}{z}Y_s(z) = 1 - \frac{z-1}{z-e^{-0.3}} = \frac{1-e^{-0.3}}{z-e^{-0.3}} = \frac{0.26}{z-0.74}$
2. $s = \frac{z-1}{T} => G_{FE}(z) = \frac{3}{\frac{z-1}{T}+3} = \frac{0.3}{z-0.7}$
3. $s = \frac{2}{T}\cdot\frac{z-1}{z+1} => G_{TUS}(z) = \frac{3}{\frac{2}{T}\frac{z-1}{z+1}+3} = \frac{0.3z+0.3}{2.3z-1.7} = \frac{0.13z+0.13}{z-0.74}$

Problem 3:

Consider the discrete-time system with state space representation

$$x_{k+1} = Ax_k + Bu_k \quad \text{where} \quad A = \begin{bmatrix} 0.1 & 0.1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$y_k = Cx_k \qquad C = [0.5 \quad -0.6]$$

1. Determine its transfer function.
2. Determine whether the system is controllable and observable (hence minimal), and stable.

1. $G(z) = C(zI - A)^{-1}B + D = \frac{0.5z+0.6}{z^2-0.1z-0.1}$
2. $Q_c = [B, AB]$, $Q_o = [C; CA]$ both have rank 2 so the system is controllable and observable and hence minimal. We can come to the same conclusion by observing that the transfer function has no common factors between numerator and denominator, so the system is minimal and therefore controllable and observable. Finally, we compute the roots of the denominator (0.37, -0.27) which are both in the unit circle, hence the system is stable.