EEE 481 Test 2 NAME: ___________________

75', Closed-book, Closed-Notes, Calculators and One 8 1/5 x 11 sheet (2pages) of notes and formulae allowed.

1. Design a PID controller to achieve a crossover of 0.5 rad/s, 50deg phase margin, for the plant with transfer function (also shown in Fig. 1)

$$P(s) = \frac{2(-s+1)}{(s+1)(s^2+2s+1)}$$

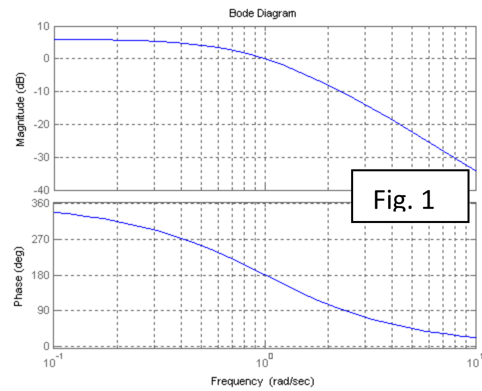

Fig. 1

Taking the PID fast pole at 10x crossover (contributing -6deg), the Phase Margin equation becomes

$$2\tan^{-1}\tau_z 0.5 - 90^{\circ} - 6 + \angle P(j0.5) = -180 + 50 \Rightarrow \tau_z = 1.45$$

(angle from zeros = 72deg), and the corresponding gain from the crossover frequency equation is K = 0.21. Thus, the PID is

$$PID(s) = \frac{0.21(1.45s+1)^2}{(0.2s+1)s} = \frac{0.43s^2 + 0.60s + 0.21}{0.2s^2 + s}$$

2. Discretize the controller for a sampling time Ts = 0.5s. Approximately, what will be the phase margin of the discrete time closed loop system?

We use Tustin for the discretization

$$PID(z) = \frac{1.32\,z^2 - 1.87\,z + 0.66}{z^2 - 0.89\,z - 0.11} = \frac{1.32\,(z-0.71)^2}{(z-1)(z+0.11)}$$

The ZOH would contribute an additional –(0.5x0.5/2) rad = -7deg to the PM equation, yielding an approximate PM for the DT system of 43deg.

3. What would be your first guess for a prefilter that attenuates step response overshoots? (Justify)

We expect some overshoot with the PID controller because the PM is less than 60deg and we are approaching the bandwidth of the RHP zero (the crossover is not higher than the open loop BW). We can start with a filter cancelling the PID zeros and with unity DC gain. Alternatively, we can use a first order filter but with a lower (0.64x) BW pole for the same effect: $F(s) = \frac{(0.2s+1)^2}{(1.45s+1)^2}$ $or,$ $\frac{(0.2s+1)}{(2.2s+1)}$

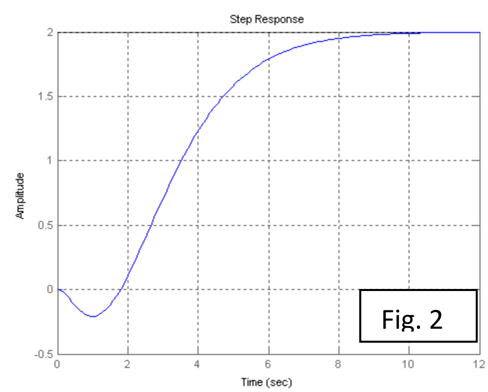

Fig. 2

4. What would be the crossover frequency below which a PI controller becomes feasible? (Justify)

The PI phase can be at most 0. Adding a practical constraint that the zero cannot contribute more than 75deg, the PI phase can be at most -15deg. Hence, at crossover, the plant phase should be at most -115 deg (+245). From the Bode plot, that corresponds to approximately 0.55rad/s. (Note: In this case, the zero is much smaller than the crossover and the PI response will be significantly worse.)

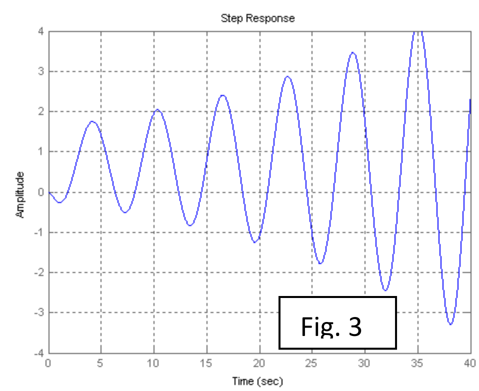

Fig. 3

4. Use Ziegler-Nichols rules to design a PI for a system with the step response shown in Fig.2.

From the graph, we estimate the slope R = 0.6 and the delay L = 1.8, yielding $PI(s) = \frac{0.85s + 0.14}{s}$. The controller is not very good because the ratio $\frac{T}{\tau} > 1$.

5. A system is tested in feedback with a proportional controller with gain 10. Use Ziegler-Nichols rules to design a PI controller for the closed-loop step response shown in Fig. 3.

From the graph, we estimate the slope Pu = 6.2 and Ku is given as ~10, yielding $PI(s) = \frac{4.5s + 0.87}{s}$.

1	P	PI	PID
Kp	1/RL	0.9/RL	1.2/RL
Ki	-	$0.27/RL^2$	$0.6/RL^2$
Kd	-	-	0.5/R

	P	*PI*	*PID*
Kp	0.5Ku	0.45Ku	0.6Ku
Ki	-	0.54Ku/Pu	1.2Ku/Pu
Kd	-	-	0.075KuPu