EEE 586, TEST 4 NAME: __ SOLUTIONS

Closed-BOOK, Closed-NOTES, ONE sheet (8-1/2 x 11) of formulae allowed, 30min

Problem 1

Consider the system $y = kG(s)u$, where the transfer function G is

$$G(s) = \frac{1}{(s+1)^4}$$

and suppose that $u = \mathrm{sat}[-y]$, where 'sat' denotes the standard saturation function in the interval $[-1, 1]$. For convenience, you may estimate the necessary quantities graphically from the included plots.

1. Use describing functions to show that the system is likely to have a periodic solution $k = 16$ and estimate its amplitude and fundamental frequency.

2. Use the circle criterion to estimate the maximum gain k for which the system is absolutely stable.

1.1 From the phase plot, the crossover frequency is $w_c = 1$, where $\angle G(j1) = -180^o$. At that frequency, $|kG(j1)| = k/4 = 4$. Then, for a periodic solution, we should have $\Psi(a) = 1/4$, for which $a = 5$.

1.2 The Nyquist plot should be to the right of the $-1/\beta$ line for absolute stability. From the plot, $k_{max} = 1/0.35 = 2.86$.

Problem 2

For the system of Problem 1, suppose that the relay nonlinearity $u = \mathrm{sign}[-y]$ is connected in the feedback instead of the saturation. Estimate the frequency and amplitude of the resulting periodic solution, if any. (D.F. $\Psi(a) = \frac{4}{\pi a}$.)

From the phase plot, the crossover frequency is $w_c = 1$, where $\angle G(j1) = -180^o$. At that frequency, $|kG(j1)| = k/4$. Then, for a periodic solution, we should have $\Psi(a) = 4/k$, for which $a = k/\pi$. This suggests that there will be an oscillation for any value of k, which is justified by the infinite gain of the relay around the origin. For $k = 16$, $a = 5.1$.

Problem 3

For the system $\dot{x} = \frac{-x}{1+x^2} + u, \quad y = x$, determine the passivity properties of the system $u \mapsto y$.

We choose $V = \frac{1}{2}x^2$. Then $\dot{V} = -\frac{x^2}{1+x^2} + xu$, or $yu \geq \dot{V} + \frac{y^2}{1+y^2}$. Hence, the system is passive. It is strictly passive and output strictly passive, but does not have a finite gain. Only in a small-signal setting, $yu \geq \dot{V} + ky^2$ with $k = \min\{1/(1+y^2)\} = 1/(1+y^2_{max})$ from which the system has an L_2 gain less than $1/k$.

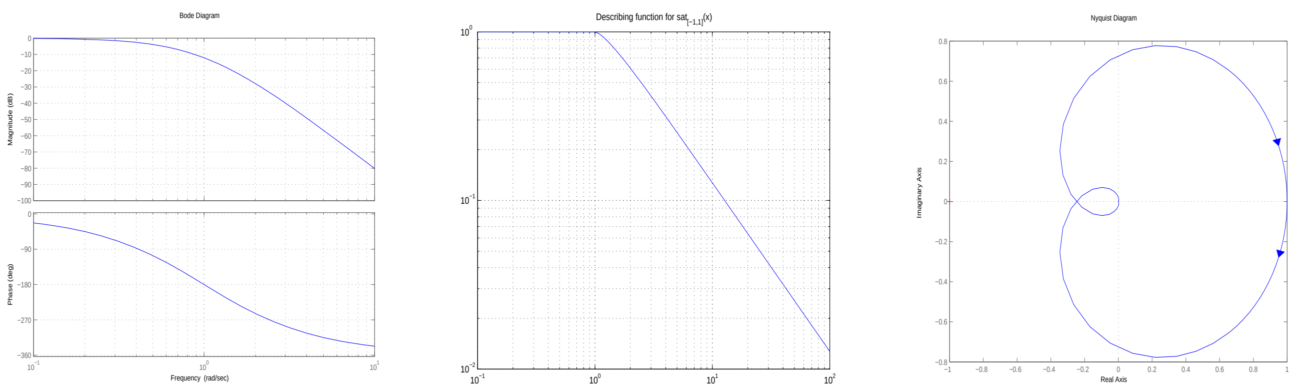

Figure 1: Bode and Nyquist plots (k = 1) and Describing Function of sat[x] for Problem 1.