EEE 582 HW 6 SOLUTIONS

33. The eigenvalues of A are (obviously) 1, 1, 1. Hence its Controller canonical form is

$$\tilde{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \text{char poly}: s^3 - 3s^2 + 3s + 1$$

We compute the contr. matrix as $\tilde{Q}_c = [\tilde{B}, \tilde{A}\tilde{B}, \tilde{A}^2\tilde{B}] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 3 \\ 1 & 3 & 6 \end{bmatrix}$

For the original system $Q_c = [B, AB, A^2B] = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$

The transformation to the cc. form $(T\tilde{x} = x)$ is

$$T = Q_c \tilde{Q}_c^{-1} = \begin{bmatrix} 4 & -4 & 1 \\ -1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

The state feedback in the cc form to assign the poles to $(-1 \pm j), -1$ (char. poly: $s^3 + 3s + 4 + 2$) is $\tilde{K} = [3, 1, 6]$, s.t. $\mathrm{eig}(\tilde{A} - \tilde{B}\tilde{K}) = -1 \pm j, -1$.

So, $-u = \tilde{K}\tilde{x} = \tilde{K}T^{-1}x \Rightarrow K = \tilde{K}T^{-1} = [-10, -33, 4]$.

For tracking step reference inputs, we want the DC gain of $[A-BK, Bp, C, D]$ to be unity. Hence,

$$p = \left. \frac{1}{C[sI - A + BK]^{-1}B + D} \right|_{s=0} = \frac{1}{C(-A+BK)^{-1}B} = \frac{1}{4}$$

34. Using the same calculations as in #33, the desired char. poly. is z^3+0z^2+0z+0, so

$\tilde{K}=[1,-3,3] \Rightarrow \operatorname{eig}(\tilde{A}-\tilde{B}\tilde{K})=0,0,0.$

Then $K=\tilde{K}T^{-1}=\underline{\underline{[10,33,-4]}}$ for which,

$\operatorname{eig}(A-BK)=0,0,0.$

The response of this system is $(A-BK)^n x_0$ with $A-BK$ being similar to its c.c. form $\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}$ which is Nilpotent and s.t. $[A-BK]^n=0$ for $n\geq 3$

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35. The subsystem $\begin{bmatrix}2&1\\&2\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}$ is c.c. In the block-diagonal form, different eigenvalues are controllable by a single input. Thus, one of the state corresponding to the eigenvalue -1 is not controllable. It follows that the eigenvalues can be placed to any locations that include -1. Hence, cases 1 & 2 are possible, Case 3 is not.

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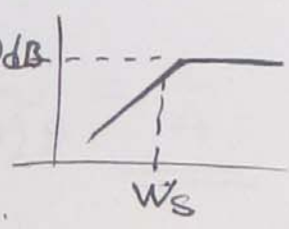
36. $K_1=[1,2]$ is such that $\lambda(A-BK_1)=(-1,-1)$

$K_{10}=[100,20]$ is such that $\lambda(A-BK_{10})=(-10,-10)$

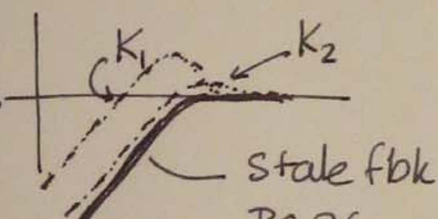
The Sensitivities have the general shape



with S-bandwidth $w_s \sim 1$ or 10 respectively. ($|S(j\omega)|<1/\sqrt{2}$ for $\omega<\omega_s$ approximately).

37. As in Pr. 36, we have $k = [1, 2]$ for the state feedback. The observer loop is also easily computed because (A, C) also happens to be the observable canonical form. We find $L_1 = [2; 1]$, $L_2 = [20; 100]$ s.t. $A - L_iC$ have the desired eigenvalues. The corresponding controllers have the st.-sp. representation $K_i = [A - Bk - L_iC, L_i, k, 0]$

Sketching the Bode plots of the sensitivities we observe a fairly large deviation from the response with the state feedback when the slow observer is used (L_1).



The L_2 observer "recovers" to an extent the state feedback properties. For general procedures and statements of recovery with high bandwidth observers (within limitations from RHP zeros) see Stein + Athans, Loop Transfer Recovery, IEEE TAC 1987.

38: Design the observer as a dual lqr problem:

$$L = (\mathrm{lqr}(a', c', b*b', R))'$$

and iterate R s.t. the observer bandwidth is as desired

(use bode(ss(a - L*c, L, -c, 1)) to find the S-BW)

We find that S-BW $\simeq 1$ for $R = 1$ ($\Rightarrow L = [1.414; 1]$)

and S-BW $\simeq 10$ for $R = 10^{-4}$ ($\Rightarrow L = [14.14; 100]$).

The controller is computed by $k = \mathrm{lqr}(a, b, c'*c, 1) \Rightarrow k = [1; 1.414]$.

Comparing with the state feedback sensitivity we observe that the two LQG controllers with fast and slow observers exhibit similar recovery properties as in Pr. 37.

EEE 582, Homework 6

Problem: A well-known difficult problem from classical feedback control is the stabilization of a transfer function with interlaced poles and zeros in the right-half plane. In this homework, we will investigate the solution of this problem using state feedback/observer methods.

Consider the system with transfer function $G(s) = \frac{s-2}{(s-1)(s-3)}$

1. Write a state-space realization for *G(s)*.
2. Design a stabilizing state feedback and an observer for your realization.
3. Compute the corresponding transfer function of the dynamic output controller *K(s)* (y -> u) and examine the root locus of *GK(s)*.

For your designs consider two cases:
1. State feedback eigenvalues at $-1 \pm j$, Observer eigenvalues $-0.2, -10$
2. State feedback eigenvalues at $-1 \pm j$, Observer eigenvalues $-1 \pm j$

1. We write a controllable (or an observable) realization to facilitate at least one of the two designs.

$$A = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [-2 \quad 1], D = 0$$

2. For the feedback we design K: eig(A+BK) are as specified.

$$A + BK = \begin{bmatrix} 0 & 1 \\ -3 + k_1 & 4 + k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \Rightarrow K = [1 \quad -6]$$

For the observer we design L: eig(A+LC) are as specified. Here, (due to low dimensionality) a "brute-force" approach works just as well as the general approach of transforming to the observable canonical form and solving the pole-placement problem there. After many straightforward computations, $\det(sI - A - LC) = s^2 + (2l_1 - l_2 - 4)s - 5l_1 + 2l_2 + 3$, which yields the following observer gains for the two cases:

1. $\det(sI - A - LC) = s^2 + 2s + 2 \Rightarrow L_1 = [-11 \quad -28]$
2. $\det(sI - A - LC) = s^2 + 10.2s + 2 \Rightarrow L_1 = [-27.4 \quad -69]$

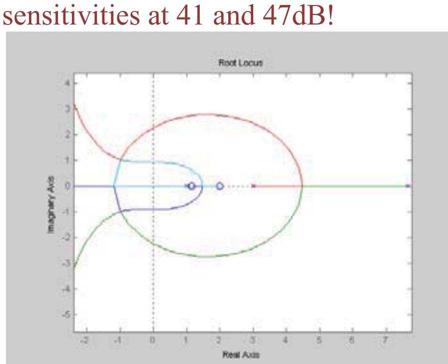
3. We compute the controller transfer functions from their state-space description:

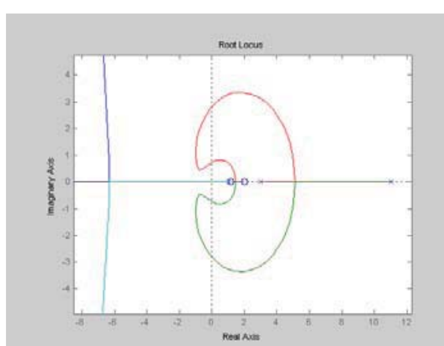
$$\dot{x}_c = Ax_c + Bu + L(Cx_c - y), \ \ u = Kx_c \Rightarrow Controller = [A + BK + LC, -L, K, 0]$$

These are in a positive feedback convention. The corresponding transfer functions are

$$C_1(s) = -\frac{157s - 182}{s^2 + 8s - 120}, \qquad C_2(s) = -\frac{386.6s - 452.6}{s^2 + 16.2s - 300.4}$$

The root-locus contains a RHP pole and a RHP zero to create the large root locus arcs needed to bring all poles in the LHP for some gains. This also results in very poor stability/robustness margins (peak sensitivities at 41 and 47dB!





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