Observer Design for Feedback Control

- ***Main issues:***

  - KF-H2-Hinf Observer Principles
  - Practical problems: Integrators and constraints
  - Objectives and Design
    - Gain selection
  - Evaluation
    - Dynamic response: injection of frequency rich signals
    - Constraint implementation: injection of large disturbances
    - Nonlinear effects: operating point transitions

  - Alternative structures

Observer Design: Kalman Filter

- Typical observers are designed as a copy of the plant with a feedback from the measurement error.

$$\dot{x} = f(x,u) \qquad \dot{\hat{x}} = f(\hat{x},u) + L(y - \hat{y})$$
$$y = h(x) \qquad \hat{y} = h(\hat{x})$$

- The feedback is designed to stabilize the error system, so that in a deterministic setting the state estimate converges to the actual state.
- For the linear case, f,h are linear functions and L is also a linear feedback.
- Also good locally for a nonlinear system and can be gain scheduled.

Observer Design: H-infinity

- In Kalman Filtering, the system is driven by noise and the objective is to minimize the state error variance:

$$\begin{aligned} \dot{x} &= f(x,u,w) \\ y &= h(x,v) \end{aligned} \qquad T_L : [w,v] \mapsto (e \doteq x - \hat{x})$$

- T_L denotes the map from the noises w,v to the state error and depends on the observer feedback L.
- The (linear) KF minimizes the H2 norm of T_L. It is optimal in a Gaussian stochastic framework.
- For feedback purposes we are more interested in the loop transfer function, e.g.,

$$T_L : [w,v] \mapsto (u)$$

- The H-inf norm provides a better setting for feedback but H2 is faster and easier to update on-line. Use H-inf as a guideline for selecting the H2 (KF) weights

Observer Design: Computations

- Linear Observer
$$\dot{\hat{x}} = A\hat{x} + L(m - C_m\hat{x})$$
$$\hat{y} = C_y\hat{x}$$
- Add deterministic inputs, if any
- Design of feedback gain L:
  - KF: min $||T_{ew}||_2$
$$AQ + QA^T + B_w S_w B_w{}^T - QC_m{}^T S_v{}^{-1} C_m Q = 0 \quad (=\dot{Q})$$
$$L = QC_m{}^T S_v{}^{-1}$$
  - Hinf: $||T_{ew}||_{inf} < g$
$$AQ + QA^T + B_w B_w{}^T - Q\left(C_m{}^T C_m - g^{-2} C_y{}^T C_y\right)Q = 0 \quad (=\dot{Q})$$
$$L = QC_m{}^T$$

  - Sv,w colored noise covariance for KF, absorbed in B,D in Hinf
  - No C_y dependence in KF (best x^ -> best y^)
  - Analogous discrete-time formulae

Observer Design for Feedback Control

- LQG: LQR+KF, Bw = Bu (disturbance at the plant input), $C_y = C_m$ (measured output)
- LTR: (observer)

$$B_w B_w^T \to I + \rho B_w B_w^T, \ \rho \to \infty$$

- Hinf: Estimate the optimal input and make the operator gain $||T_{eu}|| < g$, the same g used for control
- In the observer Riccati, $C_y = K$, the state feedback gain.
  - In this approach, the controller is designed first, the estimator uses the controller gain K, and both are connected by the same value of g.
  - Larger g revert back to the H2 solution. Often, the controller or the observer are virtually the same as LQG
  - It is not uncommon to get poor solutions when using arbitrary g values

Observer Design for Feedback Control: Comments

Constrained estimation:

- Find state estimate based on past data and subject on constraints (MPC-like)
- Find unconstrained state estimate and project on the constraint set with distance induced by the covariance matrix, preserving the Lyapunov function decay.
  - An open issue: Observers are fundamentally about getting the best estimate subject to all past info. Any change in an optimal feedback controller should aim to perturb the future optimal as little as possible.

Integrator augmentation:

- Compensating for low-frequency disturbances
- Initial weights loose meaning, (ad-hoc methods, better interpreted in terms of loop-shaping)
- State estimates may drift, constraints are no longer valid

Observer Design for Feedback Control: Examples

- Example 1:

$$\dot{x} = Ax + B_u u + B_w w$$
$$m = C_m x + 0u + D_{mw} w$$
$$y = C_y x + D_{yu} u + 0w = [C_m x; u]$$

- After augmentation (minimal order controller, at plant output)

$$\dot{x} = Ax + 0z + B_u u + B_w w$$
$$\dot{z} = C_m x + 0z + 0u - Ir$$
$$m = C_m x + 0u + Iv$$
$$y = \begin{bmatrix} C_m x + 0z + 0w \\ 0x + \ell Iz + 0w \\ 0x + 0z + Iu \end{bmatrix}$$

Observer Design for Feedback Control: Examples

- Here ℓ determines the integral action weight. Small values produce PD-like action and large values produce more integral control (default = 1)
- Alternatively, after appending with the integrator and setting the new output as z, the output matrix in the LQR objective is

$$\ell z + \dot{z} \rightarrow Q_{LQR} = C^T C + (1/\ell^2) A^T C^T C A$$

  - this has been used to get a loop shape for PID tuning, with $\ell \approx 30$
- The final controller is

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{z} \end{bmatrix} = \left(\begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} B_u \\ 0 \end{bmatrix} K - \begin{bmatrix} L \\ 0 \end{bmatrix} \begin{bmatrix} C & 0 \end{bmatrix} \right) \begin{bmatrix} \hat{x} \\ z \end{bmatrix} + \begin{bmatrix} L \\ I \end{bmatrix} (y - r)$$

$$u = -K \begin{bmatrix} \hat{x} \\ z \end{bmatrix}$$

$$L = lqr(A', C', \varepsilon I + r_{LTR} B_u B_u', I)$$

$$K = lqr(A_{aug}, B_{aug}, r_{gain} C_y' C_y, I); \quad \left(y = [Cx; z; u]\right)$$

Observer Design for Feedback Control: Examples

- Example 2:

$$\dot{x} = Ax + B_u u + B_w w$$
$$m = C_m x + 0u + D_{mw} w$$
$$y = C_y x + D_{yu} u + 0w = [C_m x; u]$$

- After augmentation (general, at plant input)

$\dot{x} = Ax + B_u u + 0v + [B_w, 0, 0]w;$ v is now the designed control

$\dot{u} = 0x + 0u + Iv + [0, I, 0]w;$ w_2 is the constant disturbance to reject, translated at the input

$m = C_m x + 0u + [0, 0, I]w$

$y = \begin{bmatrix} (C_m x + 0u + 0v + 0w)\ell \\ C_m Ax + C_m B_u u + 0v + 0w \\ 0x + 0u + Iv + 0w \end{bmatrix};$ ℓ is again the integral action weight but the output derivative must be added to provide a suitable tuning knob

Observer Design for Feedback Control: Examples

- The final controller is

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{u}} \end{bmatrix} = \left(\begin{bmatrix} A & B_u \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ I \end{bmatrix} K - LC_{aug,m} \right) \begin{bmatrix} \hat{x} \\ \hat{u} \end{bmatrix} + L(y-r)$$

$$v = -K \begin{bmatrix} \hat{x} \\ \hat{u} \end{bmatrix}$$

$$\dot{u} = v$$

$$L = lqr(A_{aug}', C_{aug,m}', \varepsilon I + B_{aug,w} B_{aug,w}' + r_{LTR} B_{aug,u} B_{aug,u}', I)$$

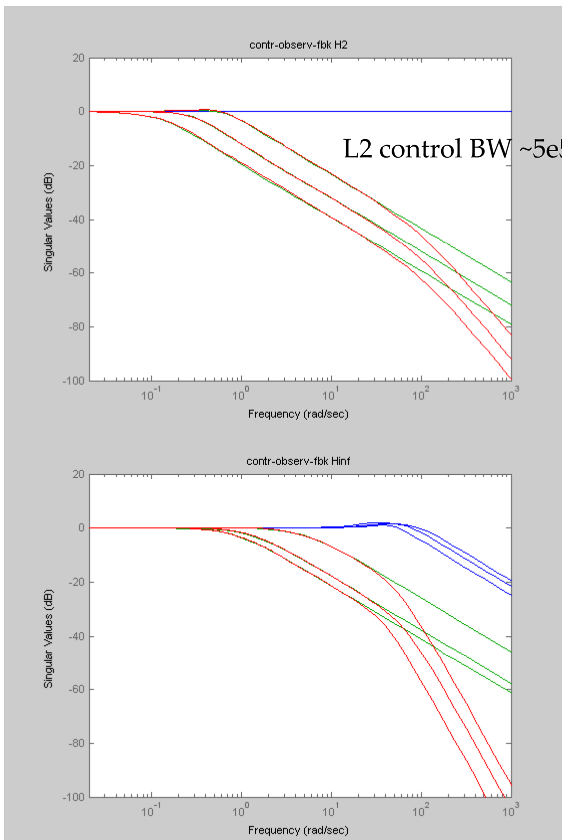
$$K = lqr(A_{aug}, B_{aug,u}, r_{gain} C_{aug,y}' C_{aug,y}, I); \quad \left(y = [Cx; C\dot{x}; u] \right)$$

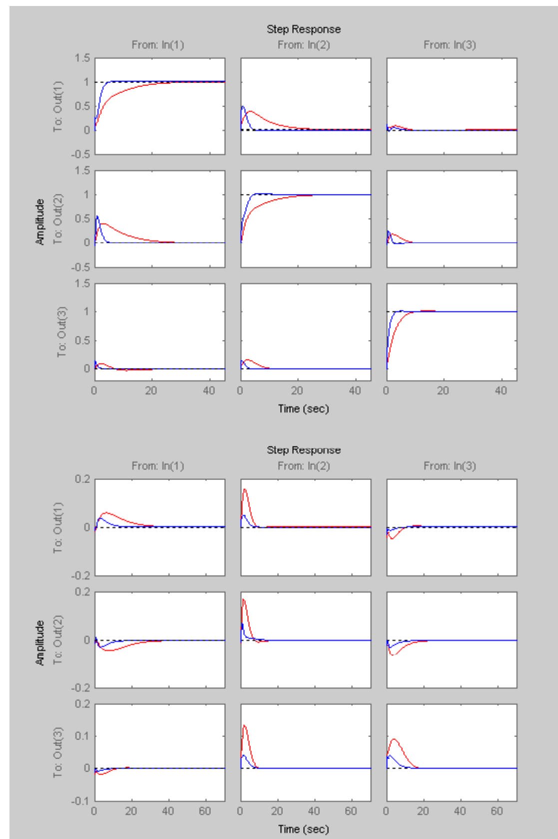
- The integrators appended to the plant now become part of the controller
- Output set-points are effectively translated at the plant input
- The integrator augmentation state must be estimated in the observer and not included as a measured state in the controller

Observer Design for Feedback Control: Examples

- Procedure:
  - Start with an H-inf design for a given r_{gain} (~ bandwidth parameter)
  - Design an LQG with similar bandwidth by changing the control weight (R)
  - Analyze the LQG observer in terms of the LTR recovery parameter
    - vary observer bandwidth
- Cases: We examine the designs for different:
  - Models (integrating or not)
  - BW (approaching RHP zero limitations or not)
- Comments:
  - For a given target BW, KF/LTR can be adjusted to a comparable performance with a good Hinf. But arbitary LTR (high or low) is not always good
  - It is desirable (for MPC implementation) to have the control BW low, but it is not always possible. Hinf with some loop-shaping considerations can provide guidance on this

H2 (top) vs Hinf (bottom) for Case 1: LTR = 1
KF dominated response, LQR BW adjustment is ineffective





Reference

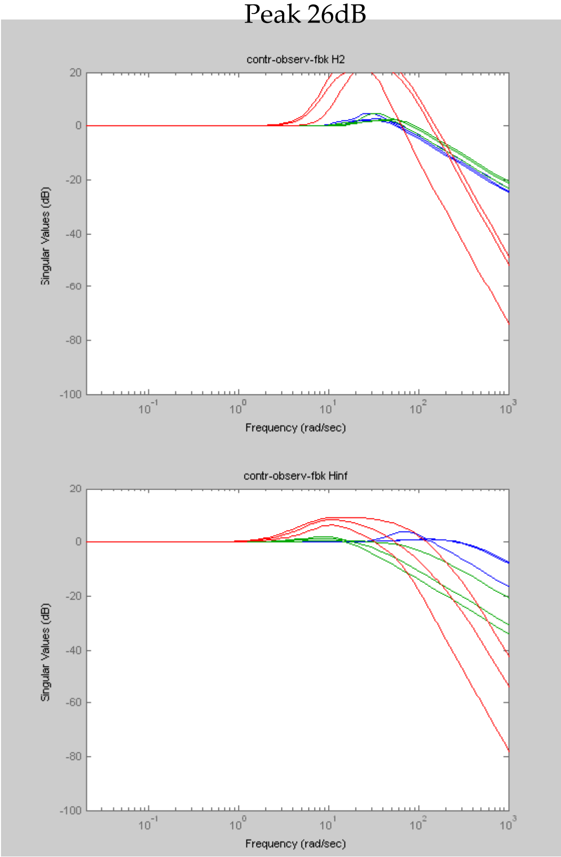
Input disturbance

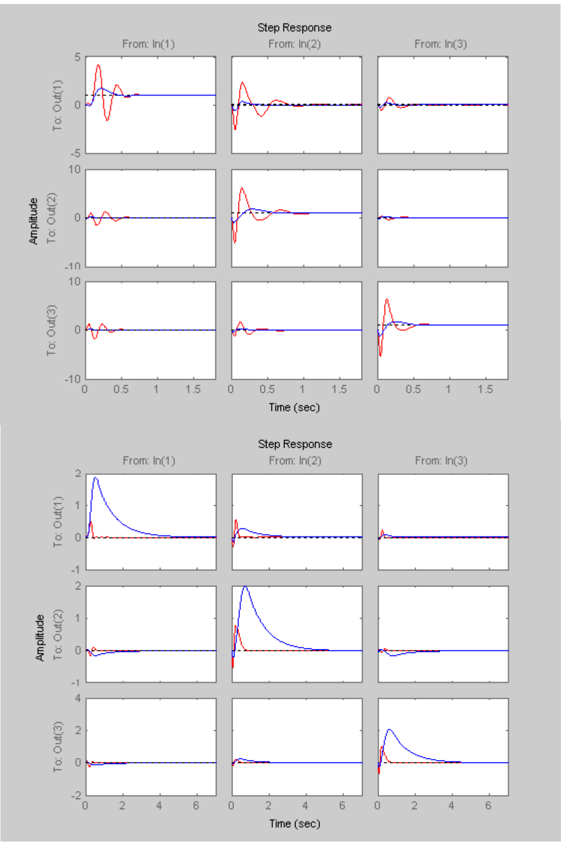
SV plots: Red-controller, Blue-LQR, Green-KF
Step resp: Blue-Hinf, Red-H2

H2 (top) vs Hinf (bottom) for Case 3: LTR = 1e6

Hinf has KF dominated response, but H2 is forced to a lower LQR BW by the high LTR parameter

Peak 26dB





Reference

Input disturbance

SV plots: Red-controller, Blue-LQR, Green-KF
Step resp: Blue-Hinf, Red-H2

Observer Design for Feedback Control: Guidelines

- Estimate target loop bandwidth BW (e.g., uncertainty data, short-term nonlinear variation)
- Use reasonable disturbance and noise models and design an LQR and a KF
  - The LQR loop is [A-BK, B, K,0] and the KF loop is [A-LC, L, C, 0]
- Evaluate the performance with LTR at the input, output, or mix
  - First metrics for comparison
    - sensitivity, co-sensitivity peaks
    - input disturbance attenuation, command tracking
    - general shape of response
  - Computation via simulation: inject a disturbance and compute signal statistics. The difference between perturbed vs. unperturbed loops yields an estimate of the corresponding sensitivity (e.g., system identification, and/or performance monitoring).

- **References: Burl, Linear Optimal Control. Prentice Hall 1998.**