EEE 582, Test 1 **NAME: __SOLUTIONS___**

Fall 2014, 30min, 1 Problem, 3 questions, Equal Credit, Closed-book, Closed-notes, calculator and 1 sheet of formulae allowed

Problem 1.

The model of a heat transfer by radiation is given as

$$\frac{dT}{dt} = -10^{-9}(T^4 - T_0^4) + \frac{V^2}{R}$$

Where T is the element absolute temperature, $T_0 = 300$ is the ambient temperature, and $\frac{V^2}{R}$ is the supplied power. Assuming that R = 2, and V is the manipulated variable:

1. Linearize the system around the constant steady-state where V = 40.
2. Write a state-space realization for the linearized model and compute its transfer function.
3. Write an approximate discretization of the state-space model with sample time 0.1sec.

1. We solve for a steady-state solution $\frac{dT_s}{dt} = 0, V_s = 40$, which for the given parameters results in $T_s = 943.3$. Next, we define the output and input variations $y = T - T_s$, $u = V - V_s$, and the state $x = y$, for which we have

$$\frac{dx}{dt} = \frac{dT}{dt} = f(T_s, V_s) + \nabla_T f(T_s, V_s)(T - T_s) + \nabla_V f(T_s, V_s)(V - V_s) + H.O.T.$$

$$\frac{dx}{dt} \simeq 0 - (10^{-9})(4)(943.3)^3 x + \frac{(2)(40)}{2} u, \qquad y = x$$

$$\frac{dx}{dt} = -3.36x + 40u, \qquad y = x + 0u$$

2. We now have $[A, B, C, D] = [-3.36,\ 40,\ 1,\ 0]$, and applying $H(s) = C(sI - A)^{-1}B + D$, we find the transfer function

$$H(s) = \frac{40}{s + 3.36}$$

3. Using Euler's discretization principle of forward derivatives we obtain the approximate discretized model as $[I + A\Delta T, B\Delta T, C, D] = [0.664,\ 4,\ 1,\ 0]$, where $\Delta T = 0.1$ is the sampling time.