EEE 582, Test 2 **NAME: ___SOLUTIONS___**

Fall 2014, 30min, Closed-book, Closed-notes, calculator and 1 sheet of formulae allowed

Problem (Questions from linear algebra):

1. Given the data points $(x_i, y_i) = [(0,\ 0.1), (1,\ 0.9), (2,\ 2.2), (3.2,\ 3)]$, find the optimum line fit $y = ax + b$ in the least-squares sense

We solve the LS problem $\min ||Ax - y||$, where $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3.2 & 1 \end{bmatrix}, x = \begin{bmatrix} a \\ b \end{bmatrix},\ y = \begin{bmatrix} 0.1 \\ 0.9 \\ 2.2 \\ 3 \end{bmatrix}$

A is full column rank so we can apply the formula $x_{LS} = (A^T A)^{-1} A^T y = \begin{bmatrix} 0.94 \\ 0.094 \end{bmatrix}$

3. Find the minimum norm solution of

$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 1 & 0 & 2 & 0 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, where x in **R**^{4}

Both equations are consistent, so it is sufficient to solve the MN problem $\min ||x||$, s.t. $[1\ \ 0\ \ 2\ \ 0]\, x = [1]$

A is full row rank so we can apply the formula $x_{MN} = A^T (AA^T)^{-1} b = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} \left(\frac{1}{5}\right)$

2. Characterize the least squares solutions of

$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 1 & 0 & 2 & 0 \end{bmatrix} x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, where x in **R**^{4}

Using elementary analysis, we observe that for all x both rows of Ax will be equal, so the solution will be such that $\min \left\| \begin{bmatrix} 1 \\ 1 \end{bmatrix} z - \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\|$. (Alternatively, we can take the linearly independent columns of A to find the LS solutions and set the remaining elements of x to zero.) We may now apply the formula so a particular LS solution is $z_{LS} = \frac{3}{2}$. Now, all the LS solutions will be such that $[1\ \ 0\ \ 2\ \ 0]x = z_{LS} = \frac{3}{2}$, or $x_{LS} = \left[\frac{3}{2}\ ; 0\ ; 0\ ; 0\right] + Null([1\ \ 0\ \ 2\ \ 0])$

4. For the triangular matrix A below, determine its Jordan form

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

The eigenvalues of A are {1,1,1,3} so the only question is how many blocks correspond to the eigenvalue 1. We compute the null space dimension of (1I-A) = 2, so there are two Jordan blocks. Hence, modulo a permutation, the Jordan form is

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$