EEE 582, TEST 5 **NAME:___SOLUTIONS____**

Fall 2014, 30min, 4 Problems, Equal Credit, Closed-book, Closed-notes, calculator and 1 sheet of formulae allowed

Problem 1. Consider a system with transfer function $H(s)$.

1. Are all realizations with the same dimension algebraically equivalent?
   No
2. Are all minimal realizations algebraically equivalent?
   Yes

If/when yes, indicate how the similarity transformation can be obtained.

The transformation and its inverse can be obtained rom the controllability/observability matrices, as $O^{+}\tilde{O}$, or $C\tilde{C}^{+}$ $\left(\tilde{A} = T^{-1}AT\right)$.

Problem 2. Consider a state space realization with dimension n and its Observability O, Controllability C, and Hankel H matrices.

1. What is the dimension of a minimal realization in terms of the ranks of these matrices?
   Dim = rank(H)
2. How is rank(H) related to rank(O) and rank (C)?
   $rank(H) \leq \min\left(rank(O), rank(C)\right)$
3. What is the dimension of a minimal realization when rank(O) = n, rank(C) = n? (Does this imply that rank(H) = n?)
   Dim = n, because the system realization is minimal. It implies rank(H) = n.
4. What is the dimension of a minimal realization when rank(O) = n-1, rank(C) = n-1? (Does this imply that rank(H) = n-1?)
   The minimal realization can have dimension n-1 or n-2. The uncontrollable mode can be the same or different from the unobservable mode. In the first case, dim = n-1, in the second, dim = n-2.

Problem 3. Given the system $u \mapsto y$: $[A, B, C, D]$, determine a state-space realization of the state feedback system $d \mapsto u$, where $u = d - Kx$ and K is a matrix gain.

[A-BK, B, -K, I]

Problem 4. Write a minimal realization of the system with transfer function $H(s) = \frac{s+1}{s^2+2s+1}$

The numerator and denominator are not coprime, hence a minimal realization would have order less than 2. After performing the cancellations

$$H(s) = \frac{s+1}{s^2+2s+1} = \frac{1}{s+1} \Rightarrow [A, B, C, D] = [-1,1,1,0]$$