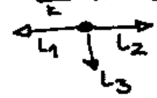
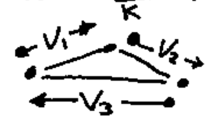
ECE 301 REVIEW NOTES

① **Power** (instantaneous) $p(t) = v(t)\, i(t)$

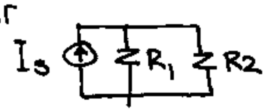
Ohm  , **Kirchoff** : KCL $\sum_k i_k = 0$  KVL $\sum_k V_k = 0$ 

② Simple Circuits

• Single loop  $i = \frac{V_s}{R_1 + R_2}$; $V_{R_2} = \frac{R_2}{R_1+R_2} V_s$

– Max Power Transfer $\max_{R_2} P_{R_2} = \frac{V_s^2}{4R_1}$ @ $R_1 = R_2$

• Single node-pair

 $V = \frac{I_s}{1/R_1 + 1/R_2}$; $I_{R_2} = \frac{1/R_2}{1/R_1 + 1/R_2} I_s$

③ Steady-state Analysis

• DC : Capacitors → Open, Inductors → Short

• AC : $V = ZI$; $V = V_0 \cos(\omega t + \theta) = V_0 \angle \theta$ { phasor notation

$I = I_0 \cos(\omega t + \phi) = I_0 \angle \phi$

$Z = |Z|\, e^{j\angle Z}$ Complex impedance

$\Rightarrow V_0 = |Z|\, I_0$, $\theta = \phi + \angle Z$.

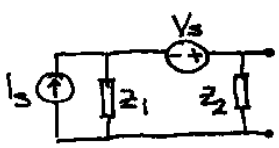
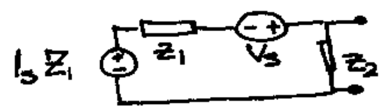
- Complex arithmetic : $a + jb = m e^{j\theta}$; $m = \sqrt{a^2+b^2}$

$\theta = \begin{cases} \tan^{-1}(b/a) & \text{if } a > 0 \\ \tan^{-1}(b/a) + \pi & \text{if } a < 0 \end{cases}$

- Impedances : Resistor $Z_R = R$, Capacitor $Z_C = \frac{1}{j\omega C}$

Inductor $Z_L = j\omega L$

- With these definitions, solve circuits as usual (nodal, loop, superposi etc)

Ex  ⟺  $I = \frac{Z_1 I_s + V_s}{Z_1 + Z_2}$

$V_{out} = Z_2 I$

Computational details: Suppose $I_s = I_0 \angle \phi_I$, $V_s = V_0 \angle \phi_V$. ← phasor

Then $V_{out} = \frac{Z_2 Z_1}{Z_1 + Z_2} I_s + \frac{Z_2}{Z_1+Z_2} V_s = \frac{Z_2 Z_1}{Z_1+Z_2}\left(I_0 e^{j\phi_I}\right) 1\angle 0 +$

complex number

$+ \frac{Z_2}{Z_1+Z_2}\left(V_0 e^{j\phi_V}\right) 1\angle 0$

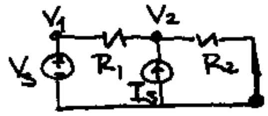
$$= \left\{ \frac{Z_1 Z_2}{Z_1+Z_2} I_0 e^{j\phi_I} + \frac{Z_2}{Z_1+Z_2} V_0 e^{j\phi_V} \right\} 1\angle 0$$

← Complex arithmetic → ⟷ time function

④ Circuit Analysis

- **NODAL:**
  - Define nodes $1 \ldots N$ and Voltages $V_1 \cdots V_N$
  - Write KCL at each node in terms of Voltages
  - Final result $Ax = b$, x = node Voltages

Ex:

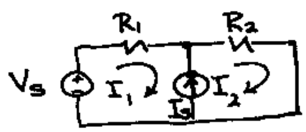


$$\left.\begin{aligned} &V_1 = V_s \\ &\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_0}{R_2} - I_s = 0 \end{aligned}\right\} \Rightarrow \begin{bmatrix} 1 & 0 \\ -1/R_1 & 1/R_1 + 1/R_2 \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} V_s \\ I_s \end{pmatrix}$$

- Special cases: Some Voltage sources → "Supernodes"
  Dependent sources → Additional variables (and equations)

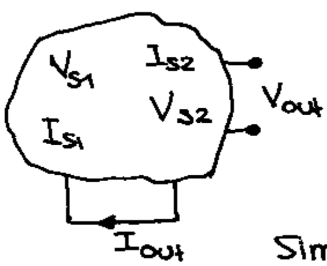
- **LOOP/MESH:**
  - Define loops $1 \ldots N$ and Currents $i_1 \ldots i_N$
  - Write KVL for each loop in terms of currents
  - Final result $Ax = b$, x = loop currents
  - Special cases: Current sources → "supermesh"
    Dependent sources → Additional variable

Ex



$$\left.\begin{aligned} &I_1 - I_2 = -I_s \\ &-V_s + R_1 I_1 + R_2 I_2 = 0 \end{aligned}\right\} \Rightarrow \begin{bmatrix} 1 & -1 \\ R_1 & R_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -I_s \\ V_s \end{bmatrix}$$

- **SUPERPOSITION:** In circuits with multiple independent sources the voltages/currents depend linearly on the sources.



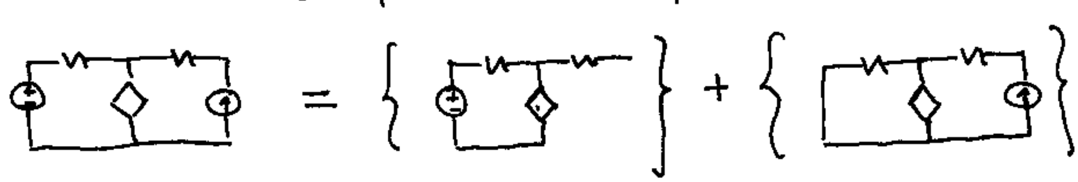
$V_{out} = f(V_{S1}, V_{S2}, I_{S1}, I_{S2})$; f : Linear function

$\Rightarrow V_{out} = \alpha_1 V_{S1} + \alpha_2 V_{S2} + \alpha_3 I_{S1} + \alpha_4 I_{S2}$; α_i = coefficien

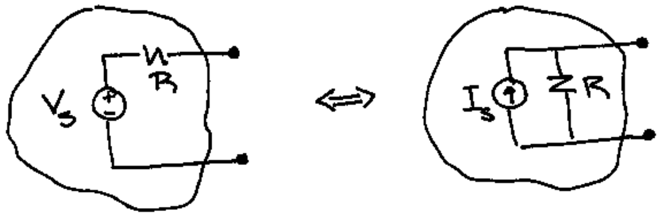
Similarly $I_{out} = \beta_1 V_{S1} + \beta_2 V_{S2} + \beta_3 I_{S1} + \beta_4 I_{S2}$.

Solution Approach:
- Keep one source at a time, replace the rest with zero sources, compute V_{out} (or I_{out}), and add up all contributions.
- Zero sources: Voltage ≡ short circuit
  Current ≡ open circuit
- Do not replace dependent sources!
  (Keep them as they are in all partial computations

Ex:



- SOURCE TRANSFORMATION

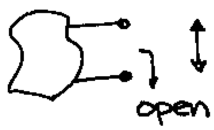


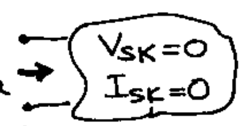
with $V_s = I_s R$

- THEVENIN - NORTON EQUIVALENTS

Two-terminal circuit

Computations:

① V_{oc} :  $V_x = V_{oc}$ ② I_{sc} :  $I_x = I_{sc}$ ③ $R_{TH} = \dfrac{V_{oc}}{I_{sc}}$ (in general)

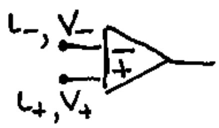
Special Cases: ■ No dependent sources $R_{TH} = R_{EQ}$ → 

$V_{SK} = 0$ → Replace voltage sources with shorts
$I_{SK} = 0$ → Replace current sources with opens

■ No independent sources : Apply a test source + compute R_{TH}.

Either current or voltage sources can be used unless the circuit is singular ($R_{TH} = 0$ or ∞); if so, one type of test source will yield R_{TH} and the other will result in inconsistent equations.

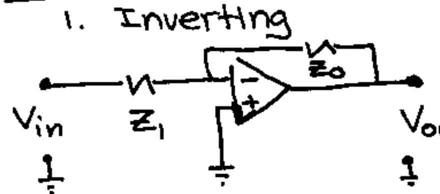
⑤ Op - Amps



$\left\{ \begin{array}{l} i_- = i_+ = 0 \\ V_- = V_+ \end{array} \right\}$ Ideal op-amp.

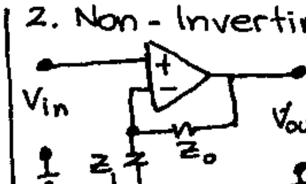
3 basic configurations

1. Inverting



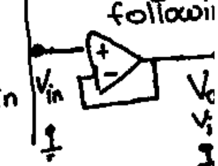
$V_{out} = -\dfrac{Z_o}{Z_1} V_{in}$

2. Non - Inverting



$V_{out} = \left(1 + \dfrac{Z_o}{Z_1}\right) V_{in}$

3. Voltage followi[ng]



⑥ Energy Storage Elements (Capacitors - Inductors)

Capacitors: $i_c = C\dfrac{dV_c}{dt} \Rightarrow V(t) = V(0) + \dfrac{1}{C}\displaystyle\int_0^t i(\tau)\,d\tau$

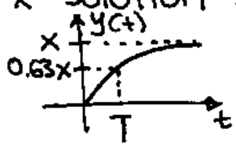
Energy stored: $W_C = \int vi = \dfrac{CV^2(t)}{2}$

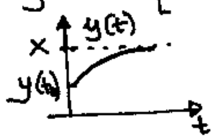
Inductors: $V_L = L\dfrac{di_L}{dt} \Rightarrow i_L(t) = i_L(0) + \dfrac{1}{L}\displaystyle\int_0^t V_L(\tau)\,d\tau$

Energy stored: $W_L = \int vi = \dfrac{Li^2(t)}{2}$

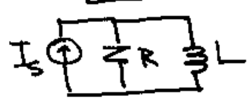
6.1 1st Order Circuits

ODE (canonical form) $T\dfrac{dy}{dt} + y(t) = x(t)$. T = time-constant

- Constant-x solution: $y(t) = e^{-\frac{t-t_0}{T}}y(t_0) + \left[1-e^{-\frac{t-t_0}{T}}\right]x$
- if $y(t_0)=0$ 

; if $y(t_0)\neq 0$ 

- TYPICAL RC CIRCUIT 

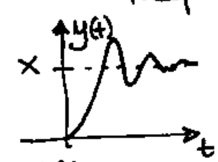
$RC\dfrac{dV_c}{dt} + V_c = V_s$; $T = RC$
- TYPICAL RL CIRCUIT 

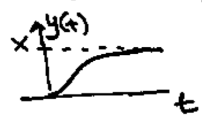
$\dfrac{L}{R}\dfrac{di_L}{dt} + i_L = I_s$; $T = L/R$

6.2 2nd Order Circuits

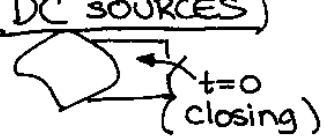
ODE (canonical form) $\dfrac{d^2y}{dt^2} + 2\zeta\omega_0\dfrac{dy}{dt} + \omega_0^2 y = x(t)$; ζ = damping ratio, ω_0 = natural undamped frequency.

- $\zeta < 1$ underdamped response
  oscillation amplitude ↑ as ζ ↓
  $\omega_0 \sim$ frequency of oscillation

  x=const: 

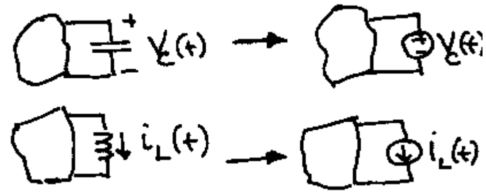
- $\zeta > 1$ overdamped response x = const. 

6.3 Circuits with switches (DC SOURCES)

(Transient Analysis) 

- KEY PRINCIPLE: Capacitor Voltages and Inductor Currents are continuous at the switching time.
  E.g. $V_c(0^-) = V_c(0^+)$, $i_L(0^-) = i_L(0^+)$

- SOLUTION PROCEDURE:
  1. Compute the solution for the continuous variables
  2. Compute the solution for the variables of interest:
     - Replace the capacitors with time-dependent voltage sources and the inductors with time-dependent current sources, and then solve the resulting resistive circuit.



① Solution for the continuous variables.

- $t<0$ steady-state $\rightarrow$ Initial conditions for the $t>0$ solution
- 1st order circuits : ODE has the form $T\frac{dy}{dt}+y(t)=y_\infty$; $y(0)=y_{0^-}$

  where T is the circuit time-constant.
  y is the variable of interest (V_C or i_L)
  y_∞ is the $t>0$ steady-state
  y_{0^-} is the $t<0$ steady-state

$\Rightarrow$ SOLUTION PROCEDURE:

(1.1) Compute the $t<0$ steady-state to find y_{0^-}.
Capacitors $\rightarrow$ open, Inductors $\rightarrow$ short, switches @ $t<0$ position:
$\rightarrow$ Solve the resistive network for y.

(1.2) Compute the $t>0$ steady-state to find y_∞.
Capacitors $\rightarrow$ open, Inductors $\rightarrow$ short, switches @ $t>0$ position:
$\rightarrow$ Solve the resistive network for y.

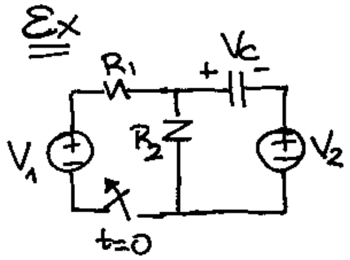
(1.3) Compute the Thevenin-equivalent resistance as seen from the capacitor / inductor terminals. Then find the circuit time constant as $T=R_{TH}C$ or $T=L/R_{TH}$

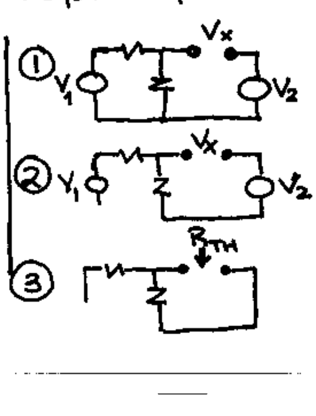
(1.4) Form the solution for $y(t)$, $t>0$: $y(t)=K_1+K_2e^{-t/T}$

$K_1=y_\infty$, $K_1+K_2=y_{0^-}$

Note: Singular cases where $R_{TH}=0$ or ∞ may require special treatment.

Ex





① $V_x=V_{C,0^-}=\frac{R_2}{R_1+R_2}V_1-V_2$

② $V_x=V_{C,\infty}=-V_2$

③ $R_{TH}=R_2 \Rightarrow T=R_2C$

$$V_C(t) = -V_2+\left[\frac{R_2}{R_1+R_2}V_1\right]e^{-t/R_2C} \quad \text{for } t>0$$