Chapter 0

Examples of System Models

0.1 Introduction

In the following, we examine the modeling of a few practical problems where feedback control is crucial. The selection of these examples is based on the type of models that describe underlying systems to be controlled (plants) and on the ability to provide simple approximations of these models. Our main objective is to expose typical modeling problems that arise in control systems design. Common features of these problems are:

- Modeling of the plant can rely on theory from diverse disciplines.
- The complexity of practical problems can increase almost without bounds, depending on the desired level of detail.
- Successful applications of feedback control (like any engineering design) hinge on making reasonable simplifying assumptions.
- Quantifying the modeling error is critical for a successful controller design (with minimal trial and error).

In particular, regarding model simplifications, it is true that very detailed models are often unnecessary. This is the beauty of feedback control as well as its main difficulty: The modeling approximations should be valid/reasonable during feedback operation. The domain of validity of the model is a question that should be resolved at the modeling step. This should then be used -in the form of specifications- to design a controller that does not cause the system to operate beyond this domain. In a sense, what we try to achieve is reasonably good closed-loop performance with the simplest possible model. The alternatives are not very appealing: Trial-and-error controller designs or extremely detailed models are time-consuming and expensive approaches.

For feedback controller design purposes, the modeling error can be quantified in a simple, theoretically sound, and practically meaningful way by estimating the spectral properties of the "multiplicative model uncertainty."[1] In a quick overview, given a system G and a model G_0, the multiplicative uncertainty is defined as a system Δ_m such that $G = (1 + \Delta_m)G_0$. (That is, it represents a percent-error in the system description.) Its spectral properties (magnitude) are then given by [2]

$$|\Delta(jw)| = |G(jw) - G_0(jw)||G_0^{-1}(jw)|$$

For computational purposes, this expression is useful when analytical models are available. Alternatively, if we apply a test input u to the system and define $y = G[u]$ as the measured (actual) output and $y_0 = G_0[u]$ as

[1] See details in the class notes *'On the notion of the "size" of a system and its applications.'*

[2] Here we assume that G and G_0 are local LTI systems represented by transfer functions. For nonlinear systems, the maximum (worst-case) should be computed over several linearization points.

the model (predicted) output, then $y - y_0 = \Delta_m[y_0]$. This simple expression suggests the following procedure for determining the spectrum of Δ_m:

1. Perform an experiment with a test input u and collect the system response data y. [3]

2. Compute the model response $y_0 = G_0[u]$, e.g., via simulation. [4]

3. Compute the Fourier transform (e.g., FFT) of the error $y - y_0$ and y_0.

4. Compute the uncertainty bound estimate $|\Delta_m(jw)| = |(y - y_0)(jw)|/|y_0(jw)|$.

A simple interpretation of this bound is very useful: *The closed-loop bandwidth should be restricted to the interval where* $|\Delta_m(jw)| < 1$.

Note: In the following examples, the units of all numerical values are in SI, unless otherwise mentioned.

0.2 Power Control System: Static model with high frequency parasitics

The delivery of a controlled and adjustable amount of power is required in a variety of applications, such as, welding, lighting, heating, motor-speed control. While voltage control is a rather expensive approach, the use of SCR/triacs has provided an efficient and cheap approach to control the current (and hence power) delivered to a load. A typical application of this circuit is as a control actuator. As such, the power is controlled indirectly, by measuring its effect on the output of interest (temperature, motor speed, light intensity).

A simplified circuit for power control using two SCRs is shown in Fig. 0.1. [5] In this circuit, two SCRs

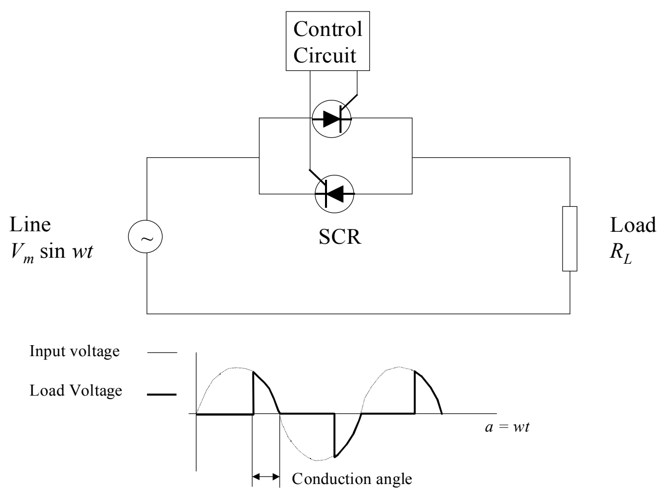

Figure 0.1: SCR Power Control circuit.

[3] The test input should contain energy in a wide range of frequencies. For nonlinear systems, u should be a perturbation around a steady-state/linearization point.

[4] Typically the model is (or is interpreted as) a local model which allows the adjustment of offsets and initial conditions. For example, suppose that (u_s, y_s) is a steady-state of the actual system and y_{meas} is the measured response to the applied input u_{appl}. Then the local model G_0 aims to predict the variation in the output $y_{meas} - y_s$ for a given variation of the input $u_{appl} - u_s$. (The model prediction of the output would be $G_0[u - u_s] + y_s$.) Looking at the variations only, we let $y = y_{meas} - y_s$, $u = u_{appl} - u_s$ and the rest of the procedure remains unchanged. Such an adjustment is consistent with the design of controllers that have integral action which guarantees zero steady-state offsets.

[5] Ref: Millman and Halkias, *Integrated Electronics: analog and digital circuits and systems.* McGraw Hill, New York, 1972.

connected in inverse parallel are used to control the negative and positive cycles of the AC power line. The SCR control circuit determines when one of the SCRs is triggered ON, connecting the line voltage to the load. The output power can be varied by controlling the phase of the conduction (conduction angle) of the SCRs. With an appropriate circuitry, the latter can be achieved by means of a voltage signal generated by, e.g., a computer. Thus, the overall power control system can be viewed as a system with input the conduction angle and output the power delivered to the load.

To derive a model for this system, we use the expression of the instantaneous power

$$P(t) = V(t)I(t)$$

where $I(t) = 0$ when the SCR is OFF and $I(t) = V(t)/R_L = \frac{V_m}{R_L}\sin wt$ when it is ON. [6] For a conduction angle ϕ, the SCR is on for the part of the cycle where $\pi - \phi < a < \pi$ and $2\pi - \phi < a < 2\pi$, where a is wt modulo 2π. [7] Hence, the average power (over a cycle) delivered to the load is

$$P_a = \frac{1}{\pi}\int_{\pi-\phi}^{\pi} \frac{V_m^2}{R_L}\left(\frac{1}{2} - \frac{1}{2}\cos 2a\right) da = \frac{V_m^2}{2\pi R_L}\left(\phi - \frac{\sin 2\phi}{2}\right)$$

For control systems design purposes, we can then write the following input-output model:

$$P_a(\tau) = \frac{V_m^2}{2\pi R_L}\left(\phi(\tau) - \frac{\sin 2\phi(\tau)}{2}\right)$$

Alternatively, with the customary notation u, y for the input and output and normalizing u to the interval $[0, 1]$,

$$y(\tau) = K\left(u(\tau) - \frac{\sin 2\pi u(\tau)}{2\pi}\right)$$

where $K = V_m^2/2R_L$.

In this expression we used τ as the time variable to emphasize the distinction between the actual instantaneous power and its average. Notice that, the power is delivered to the load in a discontinuous manner and the use of this circuit for power control relies heavily on the averaging properties of the underlying system. For example, if the power is used for heating purposes it is "obvious" that the approximation of the supplied power by P_a is sufficient to describe the bulk of the system behavior. The contribution of the fast switching of the current (60 Hz and higher harmonics) becomes negligible as it is greatly attenuated by the low-pass nature of the heat transfer. [8] In a control systems terminology, we say that the uncertainty in above simplified model of the power control system becomes large at high frequencies and hence its control objectives should be limited to low frequencies.

With this in mind, we return to the use of t as the time variable. Our power control circuit has a static (memoryless) nonlinear description

$$y(t) = K\left(u(t) - \frac{\sin 2\pi u(t)}{2\pi}\right)$$

As an additional simplification, we can attempt to provide a linear approximation of this model. Using the average slope over the entire operating region a linear model of the linear approximation of the system is just

$$y(t) = Ku(t)$$

Notice that the actual slope ranges from 0 to 2 so, locally, the difference in slopes can be quite large. Similarly, variations in Load resistance (e.g., due to temperature) will also affect the gain constant K. Any control system should therefore be able to tolerate such model differences.

[6] Here we assume a pure resistive load.
[7] Here we assume that the SCR voltage drop is negligible relative to V_m.
[8] This argument assumes a relatively "large" heating mass and specific heat.

Simulation Experiment: The file `scr_control.mdl` contains a Simulink model of the SCR Power Control System. You should study the difference between the "complete" circuit model and the averaged approximate model for different time constants of the output power filter.

Numerical values: $V_m = 100$, $R_L = 100$, $w = 2\pi 60$.

0.3 Car Cruise Control: Simple 1st-order model

Cruise control is commonly available in many modern vehicles. The system to be controlled describes the dependence of the output variable of interest (speed) on the manipulated variable (throttle position). Most of this dependence can be expressed with a straightforward application of Newton's law. However, the complexity of the model increases vastly when more detail is desired. Friction, wind resistance, and engine dynamics require considerable effort and experiments to model.

Newton's law relates the velocity of the car with the sum of the forces applied on it. Including some of the important terms (throttle acceleration (F_a), friction and wind resistance (F_f), gravity (F_g)), the model takes the form

$$m\frac{dv}{dt} = F_a + F_f + F_g$$

where m is the vehicle mass and v is the velocity.

F_a is the force applied on the car due to the torque produced by the engine. This depends on engine and gearbox characteristics, but for our purposes we will simply take that $F_a = c_a u$, where c_a is a proportionality constant and u is the throttle position.[9]

The friction term F_f is typically of the form $F_f = -c_f|v|v$ with c_f being an aggregate friction/wind resistance coefficient. Obviously, this is a gross simplification, but it is sufficient to account for the main contributions during normal operation. Changes in this term due to wind gusts or following other vehicles present perturbations that should be compensated by the control system. We will not even attempt to model the precise aerodynamic forces, although they could be very important in, e.g., racing cars.

The last term serves to describe the effect of gravitational forces when the vehicle moves on an incline. Thus, $F_g = -mg\sin\phi$ where ϕ is the angle between the horizontal and the velocity vector. This angle is treated as an unknown external disturbance for which amplitude and frequency spectrum bounds can be available.

Collecting all the terms, we obtain a model of the form

$$\frac{dv}{dt} = \frac{c_a}{m}u + \frac{-c_f|v|v}{m} - g\sin\phi$$

Next, we linearize the model around an operating point where $v = v_0$. This linearization is meaningful since we do not expect great variations in the vehicle speed when the cruise control system is engaged. Defining the output of interest as $y = v - v_0$, we have

$$\frac{dy}{dt} = -Ay + Bu + d$$

where $A = 2v_0c_f/m$, $B = c_a/m$ and $d = -c_fv_0^2/m - g\sin\phi$. Notice that, while the basic dynamics of this model are first order, A depends on the vehicle speed, B also depends on the speed indirectly through the engine characteristics (c_a) and d depends on speed and road incline. The control system should therefore adjust the throttle position u to compensate for the effect of external disturbances and tolerate variations in the model parameters.

Simulation Experiment: The file `car_control.mdl` contains a Simulink implementation of the simple nonlinear model of the car speed. Construct a linearized model and compare its behavior with the nonlinear one. (How could you estimate the model parameters using experimental measurements?)

Numerical values: $m = 1500$, $c_a = 1500$, $c_f = 0.5$.

[9] A more realistic model could be obtained by defining u to be a "commanded force" which is subsequently resolved to throttle position and brake application.

0.4 Temperature Control: From distributed-parameter to simple models

Temperature control systems form the core of many industrial applications. One of the main reasons is that physical and chemical properties of substances show a considerable dependence on temperature. First-principles modeling of a temperature control system relies on the application of energy balance principles (energy accumulation = energy in - energy out). This results in fairly complicated, usually nonlinear, partial differential equations (distributed parameter (infinite dimensional) systems). Typically, the accuracy of such models is good provided that sufficient detail has been included in the model. Such models are useful in process design and optimization but their solution requires numerical methods. On the other hand, the use of macroscopic properties allows the derivation of much simpler models that describe the bulk of the dynamic behavior of the system. Such models require the experimental determination of aggregate parameters (e.g., heat transfer coefficients) and have a relatively narrow range of validity but they are useful in control systems design.

As a first example of a temperature control system, let us consider the heating of a tubular piece of equipment. Suppose that heat is supplied to the tube at an adjustable rate q, e.g., by means of a heating coil. The tube loses energy (heat) to the surroundings at a rate proportional to the surface area times the difference between tube and ambient temperature. Finally the rate of change of the tube energy is proportional to the rate of change of its temperature. Thus, the macroscopic energy balance can be written as

$$mc_p \frac{dT}{dt} = q - h_s A_s (T - T_0)$$

where m is the mass, c_p is the specific heat (heat capacity) of the material, A_s is the exterior surface area, h_s is the heat transfer coefficient (to the ambient) and T_0 is the ambient temperature. [10] From this we see that the temperature dynamics follow a first-order low-pass model with respect to the supplied heat

$$\frac{dy}{dt} = -Ay + Bu + d$$

where u is the supplied heat normalized to the interval [0,1] (i.e., $q = q_{max} u$) y is the tube temperature and A, B, d are constants depending on the model parameters.

It should be emphasized that in reality, the temperature T predicted by this model does not necessarily correspond to the temperature of any particular point of the tube but it is an average temperature (in some sense). Furthermore, the various constants actually depend on the temperature itself as well as other external variables (e.g., pressure). Still, the model is useful in determining at least the local properties of the temperature response which are important in the design of a control system.

As a second example of a temperature control system, let us consider the previous tube with the addition of air flowing through its interior. For this, we write macroscopic heat balances for the tube and the air.

$$\begin{aligned} mc_p \frac{dT}{dt} &= q - h_s A_s (T - T_0) - h_a A_a (T - T_a) \\ \rho_a V c_a \frac{dT_a}{dt} &= -\rho_a F c_a (T_a - T_{a0}) + h_a A_a (T - T_a) \end{aligned}$$

where T_a is the air temperature inside the tube (also equal to the exit temperature), T_{a0} is the inlet air temperature, c_a is the specific heat for air, ρ_a is the air density, F is the air flow, h_a is the heat transfer coefficient from the tube to the air and A_a is the interior surface area.

Our model now takes the form

$$\frac{d}{dt}\begin{bmatrix} T \\ T_a \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} T \\ T_a \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

[10] We assume that the tube temperature and its heat transfer properties are uniform.

Depending on the case, the output variable can be either one of the two temperatures:

$$y = [1,\ 0]\begin{bmatrix} T \\ T_a \end{bmatrix}\ ,\quad \text{or } y = [0,\ 1]\begin{bmatrix} T \\ T_a \end{bmatrix}$$

In both cases, the dependence of the output on the applied power is described by a second-order model with (usually) first-order dominant dynamics that are slow relative to the intended closed-loop bandwidth.

This model contains gross simplifications: The temperature depends on the location in the tube and the air density and specific heats depend on the temperature. Avoiding the latter issue that would require simultaneous heat/momentum/mass balances, the former can be addressed by increasing the model dimension. The question here is to what extend we can consider the temperature as constant inside the tube. Performing a simple heat balance in thin slices of the tube we arrive at the model [11]

$$\begin{aligned} m_i c_p \frac{dT_i}{dt} &= q_i - h_s A_{s,i}(T_i - T_0) - h_a A_{a,i}(T_i - T_{a,i}) - kA_d \frac{(T_i - T_{i-1}) + (T_i - T_{i+1})}{\delta L} \\ \rho_a V_i c_a \frac{dT_{a,i}}{dt} &= -\rho_a F c_a (T_{a,i} - T_{a,i-1}) + h_a A_{a_i}(T_i - T_{a,i}) \end{aligned}$$

where the i subscript is used to indicate the corresponding quantity for the i-th slice; δL is the thickness of the slice; and A_d is the cross-section area of the tube. At the boundary, $T_{a,0} = T_{a0}$, the inlet air temperature. For the tube, the boundary condition is $kA_d \frac{(T_2 - T_1)}{\delta L} = h_s A_d (T_1 - T_0)$ (similarly at the other end of the tube).

Letting δL approach 0 we obtain

$$\begin{aligned} \frac{\partial T}{\partial t} &= \frac{q}{mc_p} - \frac{h_s A_s}{mc_p}(T - T_0) - \frac{h_a A_a}{mc_p}(T - T_a) + \frac{k}{\rho_s c_p}\frac{\partial^2 T}{\partial^2 x} \\ \frac{\partial T_a}{\partial t} &= -\frac{FL}{V}\frac{\partial T_a}{\partial x} + \frac{h_a A_a}{\rho_a V c_a}(T - T_a) \end{aligned}$$

where k is the thermal conductivity of the tube, ρ_s its density, and x is the distance from the inlet. The boundary conditions are $T_{t=0}, T_{a,t=0}$ =initial profiles, $T_{a,x=0}(t) = T_{a0}$, and $k\frac{\partial T}{\partial x}|_{x=0} = h_s(T_{x=0} - T_0)$, $k\frac{\partial T}{\partial x}|_{x=L} = h_s(T_{x=L} - T_0)$.

This partial differential equation model can be solved numerically by e.g., taking a finite difference approximation of the spatial derivatives (the previous model of thin slices) and solving the resulting high-order ODE. [12]

A rough analysis using the numerical values supplied below, indicates that for low flowrates (1 lit/min) the spatial distribution of the temperature has a "time-constant" in the order of centimeters. So, except for the vicinity of the end-points the temperature gradient in the tube should be small. This means that the lumped model would provide a reasonably accurate approximation of the solution. The difference between the two will be amplified if the output is measured at a particular point in the tube instead of taking the average. The accuracy of the lumped model will deteriorate further when there are uniformity deviations in the spatial distributions, e.g., abrupt changes in inlet flow/temperature. Finally, the gas heating alone has a time constant in the order of seconds. This is much faster than the tube thermal time constant $mc_p/hA_s \sim 45$min, indicating that the temperature dynamics would be essentially dictated by the heat losses.

Simulation Experiment: The file `temp_control.mdl` contains Simulink implementations of the two tube temperature models (lumped and n-point discretization of the PDE). Compare the response of two models for various conditions. [13] Also, construct a first order approximation of the 2nd order lumped

[11] These equations are derived with a perfect mixing assumption inside each slice. For low flowrates this assumption is not valid and a different derivation should be used. The difference is in higher order terms leaving the final partial differential equation the same. Notice, however, that this model is still inaccurate in that it is missing the flow equations. The more complete model would contain, not only a third PDE, but also an additional (radial) dimension.

[12] This approach may require a fairly dense spatial grid to obtain a reasonably accurate solution, depending on the values of the various parameters. In this aspect, finite element solvers have an advantage.

[13] The number of points does affect the PDE solution; for the sake of expediency you may use a low number ($\sim 100 - 200$) but you should keep it the same in all simulations.

model and discuss the properties of the modeling error. (See details in the class notes *'On the notion of the "size" of a system and its applications,'* section on Model Reduction.)

Numerical values:

Heating Element: Maximum power = 10kW.

Quartz Tube: Inner radius =0.1, Thickness = 0.01, Outer radius = 0.2 (including insulation, for the computation of heat loss area), length = 1, density = 2500, $c_p = 1078$, $k = 2$.

Air: Flowrate = 1-10 lit/min, Density = 1.2 (low temp.) - 0.5 (high temp.), $c_a = 1134$, $h_s = 5$, $h_a = 30$.

0.5 Inverted pendulum: 2nd-order unstable nonlinear model

The inverted pendulum has been a popular control experiment to demonstrate the stabilization of unstable systems. Its popularity stems from its suitability for classroom use (easy to construct, model and operate). In addition to that, there are many physical systems that exhibit similar dynamics (upright human posture, rocket control). The inverted pendulum actuator can take two different forms. One is through a torque applied directly on the pivot point (torque-pendulum), and the other is through the acceleration of the pivot point (cart-and-pendulum). Here we consider the first case. (The second is only slightly more complicated to model but significantly harder to control.)

To develop a model for this system, let us consider a rigid rod of length L, [14] supported at a free rotating pivot point, and with a mass m attached to its free end. Also, let θ be the angle between the rod and the vertical, measured counter-clockwise and with zero being at the bottom equilibrium. Torque T is applied on the pendulum rod at the pivot point, e.g., with a motor. Finally, we assume that the mass m is much larger than the mass of the rod so that the latter can be neglected.

Writing the equations of motion we have that $J\ddot{\theta} = \sum_i T_i$ where $J = mL^2$ is the inertia, $\ddot{\theta} = \frac{d^2}{dt^2}\theta$ is the angular acceleration, and T_i are the torques acting on the pendulum. In addition to the externally applied torque T, the gravity provides a torque $-mgL\sin\theta$. The friction at the pivot point also contributes a torque for which a simplified model is [15] $-c\dot{\theta}$.

Thus, we arrive at the expression

$$\ddot{\theta} + \epsilon\dot{\theta} + a\sin\theta = bT_n$$

where T_n is the applied torque normalized in the interval [-1,1], and $\epsilon = c/mL^2$, $a = g/L$, $b = T_{max}/mL^2$. This model is described by a second-order nonlinear differential equation. A typical control problem is to stabilize the pendulum in the upright position $\theta = \pi$. One approach to solve this problem is to design a controller for the linearized model of the pendulum. The linearization of the model is performed around a nominal trajectory (solution). For our case, this trajectory is $T_n = 0$, $\theta = \pi$, $\dot{\theta} = 0$. By defining u, y as the perturbations around the nominal trajectory, i.e., $u = T_n - 0$, $y = \theta - \pi$, we obtain the local linearized model

$$\ddot{y} + \epsilon\dot{y} - ay = bu$$

From the linearization theory we know that if a controller stabilizes the local linear model then it will also stabilize the nonlinear one locally, i.e., provided that the initial conditions $\theta, \dot{\theta}$ are close to the equilibrium (nominal trajectory).

Simulation Experiment: The file `pendulum_control.mdl` contains a Simulink implementation of the simple nonlinear model of the pendulum. Construct a linearized model and compare its behavior with the nonlinear one. Notice that since the equilibrium of the inverted pendulum is unstable, open-loop comparisons are meaningful for short time intervals only.

Numerical values: (Based on the GWC483 apparatus) $m = 0.275$, $L = 0.5$, $T_{max} = 0.5$, $c = 0.01$.

[14] Modeling the rod flexibility results in a distributed parameter system.

[15] The usual friction model has the form $-c|\dot{\theta}|\dot{\theta}$. This is complicated by the fact that at low anqular velocities stiction becomes important. Stiction is manifested by lack of acceleration despite the application of a small force/torque. Since a detailed model is beyond the scope of this example, we simply assume that the friction torque is proportional to the angular velocity.

0.6 Inverted pendulum on a cart

This second case of an inverted pendulum is the more traditional experiment on feedback systems design. The objective here is to keep the pendulum in the inverted position (angle $\simeq \pi$), possibly while commanding the cart to move from an initial position to a different one. This setup is significantly harder to control:

- There are two variables of concern (cart displacement and pendulum angle) and only one input (cart force).
- Torque can be applied only through acceleration of the cart. At an equilibrium the torque must be zero so that the pendulum angle can only be 0 or π. (In contrast to the previous case, here we do not issue angle commands.)
- The controller design must deal with RHP zeros that give the system its typical "inverse response." (For the cart to move to the right, it must first move to the left and unbalance the pendulum in the correct direction.)
- To the standard control input saturation, an output constraint is now added (the cart rail has finite length).

The modeling of the cart-and-pendulum system follows similar steps as the torque pendulum.[16] Suppose that a weightless rigid pendulum of length L and with a mass m at its free end is fixed on a cart of mass M. The cart moves by applying a horizontal force F on it. Define x as the horizontal displacement of the center-of-mass of the cart-and-pendulum and y that of the cart itself.

The application of the force F results in a horizontal movement described by

$$(m+M)\ddot{x} = F - c_c\dot{y}$$

where c_c is the friction coefficient for the cart motion. The cart position relative to the center-of-mass is given by

$$y = x - \frac{Lm}{m+M}\sin\theta$$

where, as usual, θ is the pendulum angle with 0 being the stable equilibrium. From this, we obtain the equation for the cart motion

$$(m+M)\ddot{y} = F - c_c\dot{y} - mL\ddot{\theta}\cos\theta + mL\dot{\theta}^2\sin\theta \tag{1}$$

Next, the pendulum rotates around its pivot point because of gravity and the acceleration of the pivot point itself. Looking at the tangential forces (generating torques), we get

$$mL^2\ddot{\theta} = -c_p\dot{\theta} - mgL\sin\theta - mL\ddot{y}\cos\theta \tag{2}$$

where c_p is the friction coefficient for the pendulum rotation.

Equations (1) and (2) describe the model of the cart-and-pendulum motion. Notice that their conversion to the standard state-space form is complicated by the fact that the highest derivative of each variable depends on the highest derivative of the other. Nevertheless, after a few manipulations, the model can be written as a fourth-dimensional ODE, with states $y, \dot{y}, \theta, \dot{\theta}$, relating the control input $u = F$ to the two measured outputs y and θ.

Simulation Experiment: The file `pend_cart.mdl` contains a Simulink implementation of the simple nonlinear model of the pendulum. Construct a linearized model and compare its behavior with the nonlinear one. Notice that since the equilibrium of the inverted pendulum is unstable, open-loop comparisons are meaningful for short time intervals only.

Numerical values: $m = 0.275$, $L = 0.5$, $c_p = 0.01$. $M = 0.5$, $F_{max} = 1$, $c_c = 0.05$.

[16] See also the textbook example, Franklin, Powell, Emami-Naeini, *Feedback Control of Dynamic Systems*, 3rd Ed., Addison Wesley, 1994.