Problem 1
In a data acquisition application we would like to use the Diamond MM board to sample 16 channels with range 0-5V, and transmit the results over the RS-232 serial port.

1. What is the minimum Baud rate required so that the transmission takes less than 0.1 sec?
2. What is the maximum error in the A/D conversion?

1. The Diamond MM has 12bit A/D so the total number of bits to be transmitted is 192. At best, we need the serial rate set higher than 1920 Baud to achieve transmission in 0.1 sec.

In a more realistic situation, each number will occupy two bytes, each using 10 bits (1 start + 1 stop) for a total of 320 bits. So the minimum rate should be 3200 Baud (or the next highest standard rate which is 4800).

2. The maximum error is \(\frac{5}{2^{10}} = \frac{5}{1024} = 0.0048828125V\).

Problem 2
The first-principles model of a temperature control system is

\[ \dot{Y} = -0.1(Y - 300) + Q, \]

where \(Y\) is the Temperature (Kelvin) and \(Q\) is the supplied heat (Watts).

1. Use the Forward Euler approximation of derivative \(\dot{Y}(t_k) \approx \frac{Y(t_{k+1}) - Y(t_k)}{T_s}\) to write a corresponding discrete time state-space model for a sampling time of 2 sec.
2. What is the discrete-time transfer function of the system?
3. What are the limitations (if any) of this discretization method.

1. Using the FE, we have

\[ Y(k+1) - Y(k) = -0.1T_s Y(k) + 30T_s + T_s Q(k) \]

There are two approaches now to derive a state space model. One is to define a new effective input \((30+Q)\) and use the temperature \(Y\) as an output and state. The other is to consider the deviations from the steady state temperature 300(K). Notice, when \(Y=300, Q=0\), then \(Y\) is constant. The latter method has better interpretations for linearization and is preferred. So, we define

\[ y = Y - 300, \quad u = Q \]

and the state \(x = y\) (first order system, one state). The above equation yields

\[ x(k+1) = (1 - 0.1T_s) x(k) + T_s u(k), \quad y(k) = x(k) + 0u(k) \]

\[ \Rightarrow A = (1 - 0.1T_s), B = T_s, C = 1, D = 0 \]

2. The transfer function (from \(u\) to \(y\), that is, from \(Q\) to \(Y-300\)) is

\[ C(zI - A)^{-1} B + D = \frac{T_s}{z - (1 - 0.1T_s)} = \frac{2}{z - 0.8} \]

3. Forward Euler is limited to sampling rates faster than the pole bandwidth to ensure that stable CT transform to stable DT system. More precisely, it should hold

\[ |1 - 0.1T_s| < 1 \iff 0 < 0.1T_s < 2 \iff \frac{1}{T_s} > \frac{0.1}{2} \]

Notice that this is not exactly the Nyquist theorem. The pole is in rad/s so if interpreted as a lowpass filter, the sampling rate should be faster than \(1/6.28\) of the corresponding Nyquist rate. It does not guarantee the ability to reconstruct signals accurately, just stability of the DT system.