EEE 304, HW 5

Problem 1:
For the feedback system shown below, compute the transfer functions $u/r$, $u/d$, $y/r$, $y/d$.

![Feedback System Diagram]

$$
\frac{u(s)}{r(s)} = \frac{C}{1 + CP}, \quad \frac{u(s)}{d(s)} = -\frac{CP}{1 + CP}, \quad \frac{y(s)}{r(s)} = \frac{PC}{1 + PC}, \quad \frac{y(s)}{d(s)} = \frac{P}{1 + PC}
$$

Problem 2: (Low Bandwidth Controller)
For the feedback system of Problem 1, suppose $P(s) = \frac{10}{0.3s + 1}$.

a. When $C(s) = K$, design $K$ so that the loop crossover frequency (i.e., $w_c$) is 0.8 rad/s. What is the contribution of a constant unit disturbance to the output?

b. When $C(s) = K(Ts + 1)/s$, design $K,T$ so that the crossover frequency is 0.8 rad/s and the phase margin (i.e., the difference between the loop angle and $-180$ at the crossover frequency, $\angle P(jw_c)C(jw_c) + 180$) is at least 60$^\circ$. What is the contribution of a constant unit disturbance to the output?

Verify in MATLAB, using $\text{step(feedback(P,K),feedback(P,C))}$

a. $\omega_c = 0.8 : \left| P(j\omega_c)C(j\omega_c) \right| = 1 \Rightarrow \left| K \right| = 0.103$

$K > 0$ for stability ($\angle C + \angle P > -180$)

$$
y_d(s) = \frac{P}{1 + PC}d(s) = \frac{10}{0.3s + 10K + 1} \frac{1}{s} = \frac{-4.93}{s + 6.77} + \frac{4.93}{s} \Rightarrow y_d(t) = 4.93 \left(1 - e^{-6.77t}\right)U(t)
$$

$$
\Rightarrow y_{d,ss} = \lim_{t \to \infty} y_d(t) = \frac{P}{1 + PC} \bigg|_{s=0} = 4.93
$$
b. \( \omega_c = 0.8 : \angle(1/s) + \angle P = -103.5^\circ > -120^\circ \)

\( \Rightarrow T = 0 \) (no zero is needed, the controller is an integrator)

\[
\omega_c = 0.8 : |P(j \omega_c)C(j \omega_c)| = 1 \Rightarrow \frac{10K}{\omega_c \sqrt{1 + (0.3 \omega_c)^2}} = 1 \Rightarrow |K| = 0.082
\]

\( K > 0 \) for stability (\( \angle C + \angle P > -180^\circ \))

\[
C = \frac{0.082}{s}
\]

\[
y_d(s) = \frac{P}{1 + PC}d(s) = \frac{10s}{0.3s^2 + s + 10K} \frac{1}{s} = \frac{10}{0.3s^2 + s + 0.82} = \frac{-7.29}{s + 1.85} + \frac{7.29}{s + 1.48}
\]

\( \Rightarrow y_d(t) = \{-7.29e^{-1.85t} + 7.29e^{-1.48t}\}U(t) \Rightarrow y_{d,ss} = \lim_{t \to \infty} y_d(t) = \left. \frac{P}{1 + PC} \right|_{s = 0} = 0
\]

**Problem 3: (High Bandwidth Controller)**

For the feedback system of Problem 1, suppose \( P(s) = 1/(0.3s + 1) \).

a. When \( C(s) = K \), design \( K \) so that the loop crossover frequency (i.e., \( w : |P(jw)C(jw)| = 1 \)) is 18rad/s. What is the contribution of a constant unit disturbance to the output?

b. When \( C(s) = K(Ts + 1)/s \), design \( K, T \) so that the crossover frequency is 18rad/s and the phase margin (i.e., the difference between the loop angle and \(-180\) at the crossover frequency, \( \angle P(j \omega_c)C(j \omega_c + 180) \)) is at least 60\(^\circ\). What is the contribution of a constant unit disturbance to the output?

Verify in MATLAB, using `step(feedback(P,K),feedback(P,C))`
a. \( \omega_c = 18 \); \( |P(j \omega_c)C(j \omega_c)| = 1 \Rightarrow \left| \frac{K}{\sqrt{1 + (0.3*18)^2}} \right| = 1 \Rightarrow |K| = 5.49 \\
K > 0 \text{ for stability } (\angle C + \angle P > -180) \\
y_d(s) = \frac{P}{1 + PC}d(s) = \frac{1}{0.3s + 6.49} \cdot \frac{1}{s + 2.16} + \frac{0.154}{s} \Rightarrow y_d(t) = 0.154 \left(1 - e^{-2.16t}\right)U(t) \\
\Rightarrow y_{d,ss} = \lim_{s \to \infty} y_d(t) = \frac{P}{1 + PC} \bigg|_{s=0} = 0.154 \\

b. \( \omega_c = 18 \); \( \angle C + \angle P = -170^\circ + \angle(Ts + 1) = -180 + PM = -120^\circ \) \\
\Rightarrow T = \tan(50^\circ)/18 = 0.066 \\
\omega_c = 1; \ |P(j \omega_c)C(j \omega_c)| = 1 \Rightarrow \left| \frac{K}{\omega_c \sqrt{1 + (0.3\omega_c)^2}} \right| = 1 \Rightarrow |K| = 1/0.016 = 63 \\
K > 0 \text{ for stability } (\angle C + \angle P > -180) \\
C = \frac{4.2s + 63}{s} \\
y_d(s) = \frac{P}{1 + PC}d(s) = \frac{s}{0.3s^2 + 5.2s + 63} \cdot \frac{1}{s + 8.67 - 11.7j} + \frac{0.14j}{s + 8.67 + 11.7j} \\
\Rightarrow y_d(t) = 2 \Re \left\{ -0.14je^{-8.67t}e^{11.7jt} \right\}U(t) \Rightarrow y_{d,ss} = \lim_{s \to \infty} y_d(t) = \frac{P}{1 + PC} \bigg|_{s=0} = 0