Pr. 19  
A. \(Q_c = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 3 \end{pmatrix}\) \(\Rightarrow\) \(\det Q_c = -1 \Rightarrow \) c.c.  
\(Q_o = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 2 & 1 \\ 1 & -2 & 1 \end{pmatrix}\) \(\Rightarrow\) \(\det Q_o = 0 \Rightarrow\) not c.o.  
B. \(Q_c = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 \end{pmatrix}\) \(\Rightarrow\) \(\det Q_c Q_c^T = 28 \Rightarrow\) c.c.  
\(Q_o = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 3 & 1 \\ 0 & 2 & -2 & 4 \end{pmatrix}\) \(\Rightarrow\) \(\det Q_o = 10 \Rightarrow\) c.o.  

Pr. 20  
It is not a well-posed problem. The given condition is necessary but not sufficient. E.g., for \((A | B) = (\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} | 0)\) is c.c. but \((\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} | 0)\) is not c.c.  
Notice that both have \((A_1, B_1)\) not c.c..  
There can be two possible fixes:  
1. \(A_{12} = 0\) then \((A_1, B_1)\) c.c. + \((A_{22}, A_{21})\) c.c. \(\Rightarrow\) \((A, B)\) c.c.  
2. \((A, B)\) c.c. \(\Rightarrow\) \((A_{22}, A_{21})\) c.c.  
For both cases, we note that controllability is equivalent to \(\text{rk} (A - sI, B) = n\) or \(\text{rk} (A_{11} - sI, A_{12}, B_1) = n\). \(A_{21}, A_{22} - sI \neq 0\) for all \(s\). Then we must have \(\text{rk} ([A_{21}, A_{22} - sI]) = n\) for all \(s\), which is equivalent to \((A_{22}, A_{21})\) c.c.  
If \(A_{12} = 0\) then \((A_{11}, B_1)\) must be c.c. and the converse will also be true (\((A_{11}, B_1)\) c.c and \((A_{22}, A_{21})\) c.c. \(\Rightarrow\) \((A, B)\) c.c.)
Pr. 21: We must have \( \det \begin{pmatrix} b_1 & a_1 b_1 + d_1 b_2 \\ b_2 & -d_1 b_1 + a_1 b_2 \end{pmatrix} \neq 0 \)

or, \( -d_1 b_1 + a_1 b_1 b_2 - a_1 b_1 b_2 - d_1 b_2^2 \neq 0 \)

or \( -d_1 (b_1^2 + b_2^2) \neq 0 \)

Equivalently \((A, B)\) is c.c. \( \iff d_1 \neq 0 \) and \( \|B\| \neq 0 \)

Pr. 22

Construct the TV controllability matrix,

\( Q_C = (B, -AB+\bar{B}) = \begin{bmatrix} 0 & -1 \\ 1 & -t \end{bmatrix} \) whose \( \det = +1 \neq 0 \)

\( \Rightarrow \) c.c.

The TV observability matrix is \( \begin{bmatrix} 0 & 1 \\ 0 & t \end{bmatrix} \) whose \( \det = 0 \)

\( \Rightarrow \) not c.o.

Pr. 23

Co. realization:

\[
[A, B, C, D] = \begin{bmatrix} (0 & 1) \\ (-2 & -3) & (0) \\ (1) & (1,1) & (0) \\ (0) & (0) & (1) & (0) \end{bmatrix}
\]

Pr. 24

There are several possible proofs, eg via eig \((A+BTC)\), contr. matrix, etc.

Here we use the definition: If \((A,B)\) is c.c. then for any \(x_0, x_f\) there exists \(\tilde{u}_0\) st. \(x_0 \rightarrow x_f\). Let \(\tilde{x}_0\) be the corresponding state trajectory.

Then the system \( \dot{x} = Ax + Bkx + Bu \) with \(u = \tilde{u}_0 \Rightarrow K\tilde{x}_0\) has the obvious solution \( x = \tilde{x}_0 \) which must also be unique by existence of uniqueness of ode solution. Hence \((A+Bk, B)\) is c.c. Similarly for the converse.